Optimization models and algorithms for downlink UMTS radio planning

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Abstract—The problem of planning third generation UMTS networks with a W-CDMA radio interface is investigated. In previous work [2]–[4] we proposed several discrete optimization models and algorithms to support decisions on where to locate base stations and which antenna configuration to select considering quality constraints for the uplink (mobile to base station) direction. The Signal-to-Interference Ratio (SIR) is considered as quality measure and we aim at a trade-off between maximizing coverage and minimizing installation costs. In this paper we extend the basic downlink model sketched in [1] and present mathematical programming models for locating directive base stations considering downlink (base station to mobile) direction and assuming a power-based as well as a SIR-based power control mechanism. The downlink direction is expected to be particularly relevant in the presence of asymmetrical traffic deriving, for instance, from data service. A randomized greedy procedure as well as a Tabu Search algorithm are adapted to find good approximate solution of the resulting NP-hard downlink BS location problem. Experimental results obtained for realistic instances with voice as well as data traffic are reported and they are compared with those provided by the uplink models and algorithms.

I. INTRODUCTION

In recent years UMTS (Universal Mobile Telecommunication Systems) systems [12] have been attracting considerable attention from a technological as well as a scientific point of view. The challenge is to enhance today’s mobile telecommunication systems (e.g., GSM) by providing increased capacity, data transmission capabilities and a wide range of new multimedia services. This goal has to be achieved by using an innovative, more flexible but also more complex, radio access scheme called W-CDMA (Wideband Code Division Multiple Access) [8].

Due to the peculiarities of W-CDMA, the UMTS radio planning problem cannot be subdivided into a coverage problem and a frequency allocation problem like it is the case for planning second generation cellular systems with a TDMA-based access scheme [9], [10], [13]. Indeed, in W-CDMA the bandwidth is shared by all active connections and no actual frequency assignment is strictly required. Moreover, the area actually covered by a base station (BS) also depends on the signal quality constraints, usually expressed in terms of Signal-to-Interference Ratio (SIR), and on the traffic distribution [5]. Since SIR values depend on emission powers, the specific power control mechanism and the power limitations must be taken into account.

In [2]–[4] we proposed several discrete optimization models and algorithms to support decisions on where to install new BSs and on which antenna configuration to select considering quality constraints for the uplink (mobile to base station) direction. We first focused on the uplink direction, since it turns out to be much more critical than the downlink (base station to mobile) direction in the presence of symmetrical traffic such as voice calls (see e.g. [8]). However, UMTS systems are also intended to provide data services which are expected to have a substantial impact on the downlink direction (e.g., web-browsing) and to yield asymmetrical traffic.

In this paper we extend the basic downlink model sketched in [1] and present mathematical programming models for locating directive BSs assuming either a power-based or a more sophisticated SIR-based power control (PC) mechanism.

The main difference between uplink and downlink models lies in the intra-cell interference. Since each BS uses orthogonal codes while different mobile stations use pseudorandom codes, in downlink the level of interference between connections involving mobiles assigned to the same BS is much lower than that due to mobiles assigned to other BSs. In an ideal environment intra-cell interference would be zero in downlink, but in practice multipath propagation implies a loss of orthogonality and hence some interference. Clearly, in the downlink direction the maximum emission power of each BS has also an impact on the installation costs.

In section II we describe the two downlink optimization models assuming a power-based and a SIR-based PC mechanism. In section III we briefly mention some aspects of the randomized greedy procedure and Tabu Search algorithm we have devised. In section IV we report some results obtained for medium-size realistic instances generated using classical propagation models. Instances including voice as well as data traffic are considered. Finally, section V contains some concluding remarks.

II. LOCATION MODELS FOR DOWNLINK DIRECTION

We assume that a set of candidate sites \( S = \{1, \ldots, m\} \) where a BS can be installed, is given and that an installation cost \( c_j \) is associated with each candidate site (CS) \( j \in S \). A set of test points (TPs) \( I = \{1, \ldots, n\} \) is also given. Each TP \( i \in I \) can be considered as a centroid where a given amount of traffic \( d_i \) (in Erlang) is requested and where a certain level of...
service (measured in terms of $SIR$) must be guaranteed [14].

The required number of simultaneously active connections for
TP $i$, denoted by $u_i$, turns out to be a function of the traffic
demand, i.e., $u_i = \phi(\bar{d}_i)$. The actual definition of the function
$\phi$ is a degree of freedom of the planning process. It can simply
respond to the average number of active connections or to
the number of simultaneous connections not exceeded with
a given probability $p$. The connection activity factor can be
considered as well.

The propagation information is also supposed to be known,
either computed by using prediction tools (e.g. Hata’s models
or ray tracing) [11]) or obtained by actual measurements.
In the case of omnidirectional BSs or of directive BSs
with a uniform horizontal diagram let $g_{ij}$, $0 < g_{ij} \leq 1$, be
the propagation factor of the radio link between TP $i$, $1 \leq i \leq n$, and a CS $j$, $1 \leq j \leq m$. The propagation
information is thus summarized by the gain matrix $G = [g_{ij}]$, $1 \leq i \leq n, 1 \leq j \leq m$.

In the W-CDMA downlink UMTS base station location
problem one wishes to select a subset of candidate sites within
the set $S$ where to install directive BSs and to assign the TPs
to the available BSs so as to maximize the traffic covered and/or
minimize the installation costs while taking into account the
signal quality requirements in terms of $SIR$ and the power
limits on the BSs. See also [16] for a discussion of various
modelling aspects.

A. Model with power-based PC

Assuming a power-based PC mechanism, the power emitted
from any BS $j$ for a connection to any given TP $i$ is adjusted so
as to guarantee a received power at TP $i$ equal to a target value $P_{tar}$. In the downlink direction, the signal quality constraint
for each connection amounts to:

$$P_{tar} / (\alpha I_{in} + I_{out}) \geq SIR_{min},$$

where $SIR_{min}$ is the minimum $SIR$ before de-spreading, $\alpha$
is the loss-of-orthogonality factor due to multipath propagation
($0 < \alpha < 1$) and the thermal noise $\eta$ is omitted as in the other
works considering a power-based PC (see e.g. [15]).

Let us define the two classes of decision variables:

$$y_j = \begin{cases} 
1 & \text{if a BS is installed in } j \\
0 & \text{otherwise}
\end{cases}$$

for $j \in S$ and

$$x_{ij} = \begin{cases} 
1 & \text{if TP } i \text{ is assigned to BS } j \\
0 & \text{otherwise}
\end{cases}$$

for $i \in I$ and $j \in S$. Suppose we consider directive BSs with
three identical 120 degree sectors and with an omnidirectional
antenna diagram along the horizontal axis. Let the index set
$I_j^2 \subseteq I$ denote the set of all TPs $i$ that fall within the sector $\sigma$
of the BS installed in CS $j$. Obviously, for each $j$, $I_j^1 \cup I_j^2 \cup
I_j^3 = I$ and the index sets $I_j^\sigma$ with $\sigma = 1, 2, 3$ are disjoint.

Since in a power-based PC mechanism $P_{tar}/g_{ij}$ is the
power that needs to be emitted from a BS in CS $j$ to guarantee
a received power of $P_{tar}$ at TP $i$, for each connection between
a BS installed in $j$ and a TP $i$ falling in a sector of this BS
the $SIR$ constraint can be expressed as follows:

$$\alpha \left( \sum_{k \in I_j^2 \cap j} u_k g_{kj} \frac{P_{tar}}{g_{kj}} - P_{tar} \right) + \sum_{k \in I_j^1 \cap j} u_k g_{kj} x_{ik} \geq SIR_{min}$$

(1)

where for any CS $l$, $1 \leq l \leq m$, the index set $I_l^\sigma$ denotes the set of all TPs in $I$ that fall within the sector $\sigma$ of the BS
installed in $l$, which contains TP $i$.

For any single connection between a BS located in CS $j$
and a TP $i$ falling in one of its sectors (denoted by $\sigma_{ij}$),
the numerator of the left-hand-side term corresponds to the
power of the relevant signal received at TP $i$ while the
denominator amounts to the total interference due to all other
active connections in the system. Indeed, the first summation
term expresses the total power received at TP $i$ that is directed
from BS $j$ to every TP $\hat{k}$ in $I_l^\sigma$, i.e., all TPs assigned to BS
$j$ and falling within the same sector as TP $i$, from which
the received power $P_{tar}$ of the relevant signal is subtracted.
The second summation term expresses the interfering power
received at TP $i$ that is directed from all BSs $l$, with $l \neq j$, to
other TPs falling within the sectors $\sigma_{ij}$ of each BS $l$ that
do also contain TP $i$. Note that, since we consider directive BSs
with uniform antennas diagram along the horizontal axis, the
propagation gains $g_{ij}$ do only depend on the distance between
TP $i$ and CS $j$ and not on the sector $\sigma_{ij}$ in which TP $i$ falls.

Thus, assuming a power-based PC mechanism, the integer
programming model amounts to

$$\max \sum_{i=1}^{n} \sum_{j=1}^{m} u_i x_{ij} - \lambda \sum_{j=1}^{m} c_j y_j$$

(2)

s.t.

$$\sum_{j=1}^{m} x_{ij} \leq 1 \quad i \in I$$

(3)

$$x_{ij} \leq \min \{1, \frac{g_{ij} P_{max}}{P_{tar}} \} y_j \quad i \in I, j \in S$$

(4)

$$\alpha \left( \sum_{k \in I_j^2 \cap j} u_k g_{kj} x_{kj} - 1 \right) + \sum_{k \in I_j^1 \cap j} u_k g_{kj} x_{ik} \geq SIR_{min} x_{ij}$$

(5)

$$\sum_{i \in I} \frac{P_{tar}}{g_{ij}} x_{ij} \leq P_{tot} y_j \quad j \in S$$

(6)

$$x_{ij}, y_j \in \{0, 1\} \quad i \in I, j \in S$$

(7)

The first term in the objective function (2) corresponds to
the total traffic covered to be maximized and the second one
to the total installation cost to be minimized. $\lambda \geq 0$ is a
trade-off parameter between these two contrasting objectives. Constraints (3) make sure that each TP $i$ is assigned to at most one BS. Note that by restricting the assignment variables $x_{ij}$ to take binary values, it is required that in every feasible solution all active connections must be assigned to a single BS. For each pair of CS $j$ in $S$ and TP $i$ in $I$, constraint (4) corresponds to the most stringent constraint among the coherence constraint $x_{ij} \leq y_{ij}$, which ensures that TP $i$ is only assigned to site $j$ if a BS is installed in $j$, and the power limit on a single connection from BS $j$ to TP $i$:

$$P_{tar} \ x_{ij} \leq P_{max} \ y_{ij} \quad (8)$$

where $P_{max}$ is the maximum emission power for the connection from CS $j$ to TP $i$ and $P_{tar}/g_{ij}$ corresponds to the emission power required by BS $j$ to guarantee the target received power $P_{tar}$ at TP $i$. For each pair of CS $j$ in $S$ and TP $i$ in $I$ the constraint (5), which is active only if TP $i$ is assigned to BS $j$ (i.e., $x_{ij} = 1$), corresponds to the signal quality requirement. Finally, constraints (6) impose an upper limit $P_{tot}$ on the total emission power of every BS.

B. Model with SIR-based PC

Let us now assume a SIR-based PC mechanism. For each connection, the emission power is adjusted so as to guarantee a signal quality level that is equal to a target value $SIR_{tar}$ [7].

As in the model with power-based PC, we consider the two classes of variables:

$$y_j = \begin{cases} 1 & \text{if a BS is installed in } j \\ 0 & \text{otherwise} \end{cases}$$

for $j \in S$ and

$$x_{ij} = \begin{cases} 1 & \text{if TP } i \text{ is assigned to BS } j \\ 0 & \text{otherwise} \end{cases}$$

for $i \in I$ and $j \in S$. But we also need to introduce an explicit continuous variable $p_i$ to denote the power received at each TP $i$ from the BS it is assigned to.

Each TP $i \in I$ can be assigned to at most one BS:

$$\sum_{j \in S} x_{ij} \leq 1 \quad (9)$$

and a TP $i$ can be assigned to a CS $j \in S$ only if a BS has been installed in $j$:

$$x_{ij} \leq y_{ij} \quad (10)$$

Since the received powers must be nonnegative and there exists an upper bound on the maximum power $P_{max}$ that a BS can assign to each connection, we have:

$$0 \leq p_i \leq \sum_{j \in S} P_{max} \ g_{ij} \ x_{ij} \quad (11)$$

Moreover, the limit on the total power that each BS $j$ can emit is accounted for by the following inequality:

$$\sum_{i \in I} \frac{p_i}{g_{ij}} \ x_{ij} \leq P_{tot} \ y_{ij} \quad (12)$$

Given the SIR-based PC mechanism, for each pair of TP $i$ in $I$ and CS $j$ in $S$ we consider the following SIR constraint:

$$p_i \ x_{ij} \leq \frac{\alpha}{\sum_{k \in I} u_k g_{kj} \ x_{kj} - p_i} + \sum_{k \in I} \sum_{\ell \in S} \frac{u_k g_{kj} \ x_{kj}}{g_{kl}} \ x_{kl} + \eta_0$$

$$\leq SIR_{tar} \ x_{ij} \quad (13)$$

which makes sure that the SIR of any active connection between BS $j$ and TP $i$ (i.e., $x_{ij} = 1$) is equal to $SIR_{tar}$. Note that equation (13) is trivially satisfied whenever $x_{ij} = 0$. As we shall see, the thermal noise $\eta_0$ plays here an important role and cannot be omitted as in the model with power-based PC.

Finally, we consider the same objective function:

$$\max_{i=1}^{n} \ s.t. \ \sum_{j=1}^{m} u_{ij} x_{ij} - \lambda \sum_{j=1}^{m} c_j y_j \quad (14)$$

III. GRASP AND TABU SEARCH ALGORITHMS

Since the two above UMTS base station location models are NP-hard [4], we have developed Greedy Randomized Adaptive Search Procedures (GRASP) and Tabu Search algorithms to find good approximate solutions in a reasonable amount of time.

GRASP is a simple heuristic in which a randomized greedy search is carried out the best available move even though it may worsen the objective function value. To prevent cycles and to try to escape from local optima, some moves are forbidden for a certain number of iterations (they are added to a Tabu list). The best solution found during the iterations is stored and returned after a predefined maximum number of steps. As initial solution of the Tabu Search algorithm we consider the solution (i.e., the set of active BSs and the assignment of TPs to active BSs) obtained with GRASP.

In the SIR-based model, if a current set of active BSs (y) and an assignment $x$ of the TPs to these active BSs are known, the corresponding value of the power $p$ received by each TP
from the BS it is assigned to can be determined as follows. Consider the SIR constraint (13) and define for each TP \(i\) the following coefficients:

\[
b_{ik} = \begin{cases} 
\alpha \left( u_k - 1 \right) & \text{if } k = i, \\
\alpha \left( u_k \frac{g_{ij}}{g_{kl}} x_{kj} \right) & \text{if } k \neq i \text{ and both TPs are assigned to the same sector } \sigma_{ij} \text{ of BS } j, \\
u_k \frac{g_{il}}{g_{kl}} x_{kl} & \text{if } k \neq i \text{ and TP } k \text{ is assigned to a sector } (\sigma_{kl}) \text{ of BS } l, l \neq j, \\
0 & \text{otherwise}
\end{cases}
\]

where

\[
\sigma_{ij} \text{ determines the received power } p_i \text{ if } k \neq i \text{ and TP } k \text{ is assigned to a sector } (\sigma_{kl}) \text{ of BS } l, l \neq j,
\]

Then the SIR constraint (13) becomes a linear equation:

\[
\sum_{k \in I'_j} b_{ik} p_k + \eta_0 = SIR_{tar} 
\]

which can be rewritten as:

\[
p_i = SIR_{tar} \left( \sum_{k \in I} b_{ik} p_k + \eta_0 \right).
\]

Discarding the SIR constraints (13) corresponding to pairs of CS \(j\) and TP \(i\) with \(x_{ij} = 0\), we obtain a linear system with one equation for each TP \(i\) in \(I\) that is assigned to a given BS, i.e., such that \(\sum_{j \in S} x_{ij} = 1\) for the given assignment \(x\). Thus, for each TP \(i\) assigned to a BS the received power \(p_i\) can be determined by solving a linear system with \(n'\) equations in \(n'\) variables, where \(n' \leq n\). By letting \(\mathbf{B}\) denote the \(n' \times n'\) matrix composed of coefficients \(b_{ik}\) and \(p\) the \(n'\)-dimensional powers’ vector, the system can be written as:

\[
\mathbf{p} = SIR_{tar} \left( \mathbf{B} \mathbf{p} + \eta_0 \mathbf{1} \right),
\]

where \(\mathbf{1}\) is the \(n'\)-dimensional vector with all 1 components. Note that the candidate values of the received powers \(p_i\) provided by the solution:

\[
\mathbf{p} = \eta_0 \left( \frac{1}{SIR_{tar}} \mathbf{I} - \mathbf{B} \right)^{-1} \mathbf{1}
\]

must still be verified. If each one of these \(p_i\) values satisfies the corresponding constraint (11), the solution composed by \(\mathbf{y}, \mathbf{x}\) and \(\mathbf{p}\) is feasible. Otherwise some TPs must be neglected (by setting \(x_{ij} = 0\) for some \(i \in I\) and all \(j \in S\)) in order to obtain a feasible solution. Since TPs that are far away from their BSs require higher emitted powers which in turn tend to generate higher interference, we try to delete first TPs that are farther away from the BSs they are assigned to.

IV. SOME COMPUTATIONAL RESULTS

Although for the uplink direction the model with power-based PC provides interesting results compared to the more accurate model with SIR-based PC (see [2], [4]), our tests show that assuming a power-based PC mechanism turns out to be totally inadequate for the downlink direction. Typical results obtained with a single iteration of GRASP for medium-size realistic instances with 881 TPs and 45 CSs are reported in Table I. Clearly, the downlink model with power-based PC yields meaningless solutions, with a very large number of BSs installed and a very low coverage, compared to those obtained with a model with SIR-based PC.

There are several reasons to account for this substantial difference. In downlink the emitted power levels for different connections from a given BS can vary much more than the different powers received by a BS in uplink, thus generating high intra-cell and inter-cell interference. Moreover, in the presence of a power-based PC mechanism the emission powers, which are selected to guarantee a \(P_{tar}\) level at the receiving end, are often much higher than needed to yield SIR values equal to \(SIR_{min}\). These oversized emission powers lead to much higher interference levels, which in turn lead to substantially worse solutions in terms of coverage as well as installation costs.

Unlike the model with power-based PC, the one with SIR-based PC is rather accurate, since the emission powers are selected in order to have a signal quality exactly equal to \(SIR_{tar}\). The interference is thus substantially lower than in a model with a power-based PC mechanism, and this leads to solutions having excellent coverage with a limited number of BSs. However, looking for approximate solutions of the downlink SIR-based model is very intensive computationally even for instances with a moderate number of TPs. Indeed, just computing the received powers \(p\) for any given choice of \(\mathbf{y}, \mathbf{x}\) involves inverting a \(n' \times n'\) matrix.

Due to the above considerations, we focus attention on the downlink model with SIR-based PC.

<table>
<thead>
<tr>
<th>Power-based downlink</th>
<th>SIR-based downlink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Served TPs</td>
<td># of BSs</td>
</tr>
<tr>
<td>526/881</td>
<td>25</td>
</tr>
<tr>
<td>570/881</td>
<td>31</td>
</tr>
<tr>
<td>597/881</td>
<td>28</td>
</tr>
<tr>
<td>559/881</td>
<td>26</td>
</tr>
<tr>
<td>596/881</td>
<td>30</td>
</tr>
</tbody>
</table>

TABLE I

RESULTS OBTAINED FOR DOWNLINK WITH THE MODELS ASSUMING POWER-BASED AND SIR-BASED PC: \(n = 881, m = 45\).

In Table II we compare the results obtained with the SIR-based PC models for downlink and uplink on realistic voice traffic instances generated according to classical propagation models (see [2], [4]). Notice that the algorithms for uplink are much faster than those for downlink. Indeed, in downlink we have to consider as many power variables (received powers
$p_k$ as there are TPs, while in uplink it suffices to consider a single received power for each BS (see [4]).

For a fair comparison between downlink and uplink, we used a number of GRASP and Tabu Search iterations for the downlink direction that requires the same computational time as the total number of iterations for the uplink direction.

Table II indicates that for instances with balanced traffic (e.g. voice calls) the uplink direction is the most stringent one from the point of view of coverage as well as of the number of BSs used. This is primarily due to the low intra-cell interference that TPs experiment in the downlink direction because each BS uses orthogonal spreading codes. Note that the downlink model yields substantial improvements: full coverage is always obtained with a very small number of BSs.

<table>
<thead>
<tr>
<th>Uplink</th>
<th>Downlink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Served TPs</td>
<td># of BSs</td>
</tr>
<tr>
<td>821/881</td>
<td>15</td>
</tr>
<tr>
<td>871/881</td>
<td>16</td>
</tr>
<tr>
<td>881/881</td>
<td>15</td>
</tr>
<tr>
<td>841/881</td>
<td>16</td>
</tr>
<tr>
<td>771/881</td>
<td>15</td>
</tr>
<tr>
<td>871/881</td>
<td>16</td>
</tr>
<tr>
<td>881/881</td>
<td>15</td>
</tr>
<tr>
<td>751/881</td>
<td>16</td>
</tr>
<tr>
<td>831/881</td>
<td>15</td>
</tr>
<tr>
<td>861/881</td>
<td>18</td>
</tr>
</tbody>
</table>

TABLE II
RESULTS OBTAINED WITH SIR-BASED PC MODEL FOR UPLINK AND DOWNLINK: $n = 881$, $m = 45$.

Since UMTS is intended to provide, especially in the downlink direction, not only voice but also data services (e.g. web-browsing), we have considered instances with both types of traffic. To distinguish voice and data traffic, we have added 176 TPs with $SIR_{min} = 12dB$ (which corresponds to a spreading-factor equal to 32) to the other 881 TPs with $SIR_{min} = 6dB$. Table III reports the results obtained with a single iteration of GRASP applied to the downlink model with the above-mentioned type of instances. A larger number of BSs is clearly necessary to guarantee a good coverage.

<table>
<thead>
<tr>
<th>Served TPs</th>
<th># of BSs</th>
</tr>
</thead>
<tbody>
<tr>
<td>841/1057</td>
<td>15</td>
</tr>
<tr>
<td>796/1057</td>
<td>17</td>
</tr>
<tr>
<td>855/1057</td>
<td>15</td>
</tr>
<tr>
<td>845/1057</td>
<td>18</td>
</tr>
<tr>
<td>716/1057</td>
<td>12</td>
</tr>
</tbody>
</table>

TABLE III
RESULTS OBTAINED WITH SIR-BASED PC MODEL FOR INSTANCES WITH VOICE AS WELL AS DATA TRAFFIC (TPS HAVING DIFFERENT $SIR_{min}$): $n = 1057$, $m = 45$.

V. CONCLUDING REMARKS

We have presented two optimization models for locating BSs in UMTS networks taking into account the downlink direction and assuming either a power-based or a SIR-based PC mechanism. GRASP and Tabu Search algorithms have been proposed to find approximate solutions of the above models within a reasonable amount of computational time. Unlike for the uplink direction, in downlink the more accurate model with SIR-based PC is required to obtain meaningful solutions. For instances with mixed voice and data traffic, it is expected that the uplink model provides a good estimate of the number of BSs that are actually needed while the downlink model of the amount of traffic that can be covered. The results obtained with the combined model including uplink [2], [4] SIR-based as well as downlink SIR-based constraints 13, which cannot be presented here for lack of space, indicate that by starting from the solutions provided by the simplified uplink power-based model [2], [4] computing times (which are of the order of days) are reduced by more than a factor of fifty without affecting the solutions’ quality.

REFERENCES