Aliasing and irregular sampling for Kirchhoff integral operators
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Summary

Algorithms for quantitative analysis of seismic data have to be amplitude consistent. Kirchhoff implementation of seismic operators is widely used for its ability to handle irregularly sampled data. However, the so-called true amplitude kernels fail to be strictly amplitude consistent when data sampling is irregular. The method proposed here for data weighting allows the reduction of artifacts introduced by irregular sampling. The presented approach can be seen as an extension of the well-known dip equalization [3], and it can be applied to all Kirchhoff-type operators. Moreover this approach allows to investigate the operator aliasing issue even in presence of irregular sampling.

The potentialities of the method are proved by showing the improved results obtained while using the SCO integral operator (Common Shot Continuation) for 3D irregularly sampled data.

Introduction

3D seismic surveys often have coarse and irregular acquisition geometries. With such datasets, Kirchhoff type integral formulas show their limitations in amplitude behavior. These limitations arise even if the acquisition geometry is regular, because this condition does not imply regular sampling over Kirchhoff summation surfaces: the fundamental concept of analyzing sampling in dip domain was first introduced by Beasley and Klotz [3]. So, the minimization of the acquisition footprint turns out to be an important topic for operator implementation, as well as the aliasing control.

Irregular sampling and trace weighting

The limitations of Kirchhoff type methods in presence of irregularly sampled data are more related to the approximations used in the numerical evaluation of Kirchhoff surface integral (that means the discretization of a space-continuous expression) than to limitations in the derivation of the integral formulas themselves. So a careful approach to the approximation of the continuous integral expression with a discrete sum can improve operator results and remove some artifacts due to the non-correct destructive interference.

It is important to point out that Kirchhoff curve (or surface) sampling must be analyzed in its natural domain, that is \((\phi, \alpha)\) domain (where \(\phi\) represents event dip, and \(\alpha\) azimuth) as already stated in [3],[4]. While working in this domain, we implicitly handle both acquisition geometry irregularities and illumination variations due to wavefield distortion through complex media. When the sampling in the \((\phi, \alpha)\) domain is regular, the result of integration \(I_K\) (that is the estimation of DC component of the signal) of \(N\) samples of signal \(s_n\) is:

\[
I_K = \frac{1}{N} \sum_{n=1}^{N} s_n
\]

When the sampling is irregular, the summation without any weighting or equalization causes artifacts.

The weighting scheme derived here is based on a least square approach to the (implicit) reconstruction of regularly sampled data from the irregularly spaced traces. In fact, irregularly sampled data \(s=[s_1, \ldots, s_N]^T\) can be expressed as the interpolation of \(M\) regularly sampled data \(m=[m_1, \ldots, m_M]^T\) (with \(M \leq N\)) to an irregular grid by using any interpolation matrix \(A\). Regular samples can be obtained with a LS solution of the system:

\[
\begin{bmatrix}
  s_1 \\
  \vdots \\
  s_N
\end{bmatrix} = A \cdot 
\begin{bmatrix}
  m_1 \\
  \vdots \\
  m_M
\end{bmatrix}
\]
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Linking together the two previous relationships it leads to the following expression of weights $w$ that minimizes the irregular sampling artifacts:

$$I_k = \frac{1}{M} \sum_{i=1}^{M} m_i$$

$\begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} = A \cdot \begin{bmatrix} m_1 \\ \vdots \\ m_M \end{bmatrix} \Rightarrow I_k = \frac{1}{M} \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \left[ (A^T A)^{-1} A^T \cdot s \right] = \begin{bmatrix} w_1 & \cdots & w_N \end{bmatrix}$

The accuracy of the method and its computational cost depend on the length of the interpolation functions, implicitly used in the matrix $A$. It is easy to show that the choice of a first order interpolation scheme (nearest neighborhood), corresponds to the operator equalization first proposed by Beasley and Klotz [3] for the DMO. In fact, when nearest neighborhood interpolation is used, matrix $(A^T A)^{-1}$ is diagonal (and its inverse is trivial) and exactly corresponds to dip equalization. Using higher order interpolators (i.e. linear, cubic) allows to obtain better results, as shown in figure 1, while increasing the computational costs to find weights $w$. For the other examples shown in this paper a cubic interpolation scheme is used as it represents a compromise between accuracy and computational cost. The spacing of regularly sampled model $m$ is imposed by the existence of the inverse of the matrix $(A^T A)$, so it is related to the choice of the interpolating function, too. While it is straightforward in 2D seismic, choice of model spacing is not simple for 3D seismic.

Aliasing condition for irregular sampling

Effective anti-aliasing filtering is essential in obtaining well-imaged results [1], [2]. Aliasing of Kirchhoff type operators does not arise if and only if dips (plane waves) that should interfere destructively continue to interfere destructively (even if they are aliased). If $\Delta p$ is defined as the maximum dip difference between operator and data ($\Delta p = \max(|p_{\text{min}} - p_{\text{op}}|, |p_{\text{max}} - p_{\text{op}}|)$ for each spatial axis $i$, a sufficient condition to avoid operator aliasing for 3D regularly sampled seismic data is:

$$\text{Spectrum} \left( f \geq \frac{1}{\max(\Delta p, \Delta \Delta)} \right) = 0, \quad i = 1, 2;$$

However it is important to point out that this is not a necessary condition. For each data dip, there are only a few time frequencies that effectively contribute to alias (and this happens only if the particular dip is really present in data): the previous equation can be written as an equality (as simply shown in figure 2). In principle, if data dips are known, Kirchhoff operators that can image “beyond aliasing” can be designed. Then, this allows to explain why sometimes the use of anti-alias filtering produce a too low-passed (i.e. poor quality) seismic section, while we can obtain better results with no anti-alias filtering applied, especially in 3D seismic.

A main advantage of the previously derived expression for data weights is that the anti-aliasing condition can be related to irregular sampled data by the link of the implicit regularly sampled model $m$. The use of a higher order interpolating function permits to relax anti-aliasing condition more than the simple summation. In fact, when the simple summation (or nearest neighborhood interpolation) is used, the existence of the inverse of $(A^T A)$ imposes $\Delta m$ (spacing of regularly sampled model) to be equal to the maximum $\Delta s$ (spacing of irregularly sampled data); choosing longer interpolators, $\Delta m \leq \max(\Delta s)$. So, relaxing aliasing condition by the developed weighting method improves the output sections.

Application to the Shot Continuation Operator

We show results obtained with a particular Kirchhoff-type integral operator: the 3D shot continuation operator. The 3D SCO is a pre-stack operator for 3D acquisition geometries that estimates a common shot gather (CSG) from neighboring CSGs; this estimation is performed according to a specified velocity model [5], [6]. 3D SCO can be specialized in any domain (e.g., offset and azimuth) and it can be viewed as a generalization of the DMO (or continuation to zero offset). As a pre-stack operator, the minimization of the artifacts introduced by irregularities usually encountered in real data allows reliable pre-stack analyses, such as AVO, for the interpolated data.
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The example shown here is part of a synthetic marine like geometry (part of SEG/EAGE M3D modeling project); geometry is made up of eight receiver lines (distance between lines is 50 m) 4000 meters long, while shot spacing is 100 m. After 1:2 shot decimation, every missing shot has been reconstructed by the two neighboring ones (shot distance: +/- 100 m). Irregular sampling is obtained by simulating 40% of dead hydrophones for the input CSGs (the map of dead traces is shown in figure 3). The interpolated data show better behavior both by quantitative (MSE is plotted in figure 3) and qualitative inspection when the equalization method developed in this paper is used.

Conclusions

In this paper a method to obtain various weighting schemes to apply to irregularly sampled data is developed, in order to minimize amplitude artifacts due to non-uniform sampling. The described approach contains the well-known dip-equalization approach, first described in [3], it’s dip independent and permits to understand aliasing condition for irregularly sampled data. Moreover, it can be applied to any Kirchhoff-type integral operator. The improved results obtained by the correct weighting are verified by the results shown with 3D SCO operator.

References


FIGURE 1. 10 irregularly sampled panels are stacked (3 examples are shown on the left): stacking must conserve only the horizontal reflection. Summation artifacts are plotted on the right, for 3 different choices of the interpolating function. Each trace represents stacking error for one panel. Error amplitudes are magnified by a factor of 30.
FIGURE 2. This simple example of stacking one aliased dipping event shows how only one frequency contributes to aliasing noise: operator aliasing occurs exactly when \( f = (\Delta p \cdot \Delta s)^{-1} = (0.96 \text{ ms} / \text{m} \cdot 25 \text{ m})^{-1} \cong 42 \text{ Hz} \).

FIGURE 3. The result of 3D SCO reconstruction is improved by the use of the equalization proposed in this paper (a cubic interpolator is used to determine weights): the two common shot panels represent the result of 3D SCO reconstruction, without (on the left) and with (on the right) equalization. Mean square error of the equalized operator (blue line) is smaller than the original one (red line), especially in correspondence of groups of dead receivers. The two contributing shots are at DS=\(+/-100\) m.