VARIABLE RANK RECEIVER STRUCTURES FOR LOW-RANK SPACE-TIME CHANNELS

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Abstract – MMLSE is known to be the most suitable approach for space-time (S-T) receivers that requires the knowledge of the S-T channel response. In general we do not have the knowledge neither of the channel \( H \) nor of co-channel interference’s (CCI) statistics, these have to be estimated by using training sequences. When short data preambles are available the large variance of the unconstrained estimate of the multichannel \( H \) can heavily affect the achievable performance. This paper illustrates the advantages of a low rank truncation of the whitened channel in a mobile radio scenario where the S-T diversity of the channel is almost limited. The benefits achievable with this class of low-rank receivers are the reduction the variance of the multichannel response estimate and the lower complexity compared to a full-rank solution. The performances in realistic scenarios demonstrate the advantages compared to conventional receivers.

I. INTRODUCTION

Space division multiple access represents a useful approach to increase the capacity of modern radio communication systems. This can be effectively achieved by employing an array of antennas/receivers \([1][2]\) at the base stations as this allows to reduce the frequency reuse distance. However, the two major problems that have to be faced are mainly related to the trade-off between spectral efficiency and receiver accuracy: 1) the length of the training sequence (preamble) that is used for channel estimation and equalization must be kept short, therefore the multichannel estimates tend to be noisy (i.e. with high variance); 2) the co-channel interference (CCI) is asynchronous and therefore “attractive” performances can be obtained only if the receiver can cope with time-varying interferences, this latter problem is often ignored in the literature. Based on the analysis of the properties of the space-time (S-T) channel \([3]\), we propose here a reduced-complexity and fast-convergent S-T receiver scheme useful for the CCI rejection in TDMA systems (e.g., GSM/DCS system).

II. MODEL AND PROBLEM DEFINITION

In a multipath environment the spatial features of the channel caused by the relative positions between the array elements and the MSs, heavily affect the shape of the overall channel response. Such a response can be conveniently treated as a random process. A commonly agreed model under no-LOS conditions is the Gaussian Wide-Sense Stationary uncorrelated scattering. We consider an \( M \) sensor array located in the base station (BS); the channel response for each sensor is given by the convolution of the physical channel and the responses of the transmitting and receiving filters. The base-band equivalent channel of the signal received by the \( k \)-th array element is \( \hat{h}_k(t) \). All the signals received by the array can be conveniently arranged into an \( M \)-dimensional vector

\[
y(t) = \sum_{n=\infty}^{+\infty} x_n \hat{h}(t-nT),
\]

\((1)\)

\(\{x_n\}\) is the transmitted sequence. In general, more independent sources (users) are present (or can be considered to have a meaningful power level) in a cellular system with low reuse distance. The signal received by the antenna array will be

\[
y(t) = \sum_{n=\infty}^{+\infty} x_n \hat{h}(t-nT) + \sum_{q=1}^{Q} \sum_{n=\infty}^{+\infty} i_{n,q} \hat{h}_q(t-nT) + n(t)
\]

\((2)\)

where \(\{i_{n,q}\}\) are the interferers’ sequences and \(n(t)\) is spatially and temporally uncorrelated Gaussian noise \((E[n(t)n^H(t+\tau)] = \sigma^2 I(\tau))\).

By sampling the received signal and collecting \(N\) time samples of the burst into a single matrix, the received signal \((2)\) can be rewritten by using stan-
standard matrix notation:
\[
\mathbf{Y} = \mathbf{HX} + \sum_{q=1}^{Q} \mathbf{H}_q \mathbf{I}_q + \mathbf{N};
\]
\[
\mathbf{Y} = [\mathbf{y}(T), \ldots, \mathbf{y}(NT)] \in \mathbb{C}^{M \times N} \text{ are } N \text{ samples (symbol sampling interval) of the signals received by } M \text{ antennas and } \mathbf{X} = [\mathbf{x}(T), \ldots, \mathbf{x}(NT)] \in \mathbb{C}^{L \times N} \text{ the asymmetric Toeplitz matrix of } (N+L) \text{ transmitted symbols } (L \text{ is the channel memory}), \mathbf{H} \in \mathbb{C}^{M \times L} \text{ denotes the multichannel and } \mathbf{N} = [\mathbf{n}(T), \ldots, \mathbf{n}(NT)] \text{ is the additive noise and interference.}
\]

III. MAXIMUM LIKELIHOOD SEQUENCE DETECTORS

Gaussian interference

The actual mobile radio systems are known to be interference limited, that is the noise is a structured non-Gaussian signal generated by mobile stations that use the same bandwidth resource of the desired user. A truly optimum (maximum likelihood) solution would be a multi-user receiver, i.e., all active users are detected jointly. It is nevertheless recognized that the Viterbi receiver is optimum in a least square sense, or equivalently it is the MLSE when the interference can be approximated as Gaussian.

The MMLSE detector jointly exploits the spatial and temporal characteristics of the multipath channel to estimate the most likely transmitted sequence given the received signal. Therefore it is only applicable to the reverse link (BTS-MS) of the communication system. The minimization of the negative log likelihood is performed over the alphabet \( \mathcal{A} \) of admissible symbols specified by the modulation format and it can be effectively carried out by means of the linear programming techniques (i.e., Viterbi algorithm)

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{A}} \| \mathbf{Y} - \mathbf{HX} \|_F^2
\]

The metric computed for the paths along the trellis of the demodulator results from the sum of the metrics computed on the different subchannels (one for each antenna):

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{A}} \sum_{i=1}^{M} \| \mathbf{y}_i - \mathbf{h}_i \mathbf{X} \|_F^2
\]

where, for the \( i \)-th antenna, \( \mathbf{y}_i \) is the vector of the \( N \) received samples and \( \mathbf{h}_i \) is the (known) temporal channel. The processing (or modulation/demodulation) scheme must take into account noise correlation (spatial structure). In order to do this, it is enough to embody a space decorrelator to reject the spatially correlated noise before the standard MMLSE \( [3] \), that is the norm must be properly weighted with the corresponding covariance matrix \( \mathbf{Q} \)

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{A}} \| \mathbf{Y} - \mathbf{HX} \|_Q^2
\]

The metric has been simplified here by considering temporally uncorrelated CCI.

Reduced-rank metric

The MMLSE detector uses the metric computed on different branches of the receiver. However, since the effective rank of the multichannel is almost limited, the metric can be computed only on the useful dimensions. By defining the prewhitened signal \( (\mathbf{Y}' = \mathbf{Q}^{-1/2} \mathbf{Y}) \) and channel \( (\mathbf{H}' = \mathbf{Q}^{-1/2} \mathbf{H}) \) the MMLSE becomes:

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{A}} \| \mathbf{Y}' - \mathbf{H}' \mathbf{X} \|_F^2
\]

Since the Frobenius norm is invariant respect to unitary transformations \( [9] \), we may rewrite the MMLSE \( (5) \) with respect to another reference system on an orthonormal basis \( \mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_M] \):

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{A}} \sum_{n=1}^{M} \| \mathbf{u}_n^H \mathbf{Y}' - \mathbf{u}_n^H \mathbf{H}' \mathbf{X} \|_F^2
\]

The multidimensional metric is the sum of the metrics in the 1-D orthogonal subspaces of \( \mathbf{U} \). In other words, when reconsidering the MMLSE receiver's structure there are \( \min(M, L) \) beam-formers \( (\mathbf{w}_n^H = \mathbf{u}_n^H \mathbf{Q}^{-1/2}, \text{see figure 1}) \) each one with a matched filter (MF) after the beam-former \( \mathbf{h}_n^H = \mathbf{w}_n^H \mathbf{H} \). Since the whitened channel matrix \( \mathbf{H}' \) is low rank, say \( r \), the receiver structure may be simplified (i.e., the number of beam-former can be reduced) by choosing the proper natural basis \( (\mathbf{U}_r) \) such as: \( \text{span}(\mathbf{U}_r) = \text{span}(\text{eig}(\mathbf{U}_r, \mathbf{Q})) \). By dividing the whole space into the signal subspace \( \mathbf{U}_r \) and the (complementary) noise subspace \( \mathbf{U}_{r}^\perp \) (note that \( \mathbf{U} = [\mathbf{U}_r, \mathbf{U}_{r}^\perp] \)), the whole metric may be expressed as:

\[
\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} \sum_{n=1}^{r} \| \mathbf{w}_n^H \mathbf{Y}' - \mathbf{h}_n^H \mathbf{X} \|_F^2
\]

here \( r < \min(M, L) \). The structure of the overall receiver is in figure 1.

Analysis of the rank of \( \mathbf{H} \) (GSM air interface)

Since the propagation channel \( \mathbf{H} \) can be estimated only according to the signal bandwidth and array
aperture, $H$ is almost low-rank, or equivalently it can be approximated by a combination of few orthogonal sub-channels: $H \approx \sum_{i=1}^{r} u_i h_i^*$, the channel-rank is $r < \min(M, L)$ (the low-rank nature of $H$ is due to the limited angle or delay spread of the multipath channel compared to the array and delay resolution, for the detailed analysis we refer to [5]). According to the low-rank model, the channel estimation can be limited to only few orthogonal rank-one channels.

In order to gain some insight into the low-rank nature of the multichannel, it is useful to evaluate disjointedly the spatial and temporal dependency of the residual of the rank-$r$ approximation of the multichannel conditioned to the power delay profile of ETSI model and to the azimuthal power distribution. Due to the non-linearity of the analysis, the evaluation was carried out through MonteCarlo simulations. For the TU GSM case (here we refer to the ETSI channels and GSM air interface, see also [4]) and moderate angular spread (7.5 deg) the multichannel is almost rank-2 (figure 2). For an ULA with 8 $\lambda/2$ spaced elements, the time diversity is the limiting factor (this does not hold when the signal bandwidth increases as in this case the space diversity becomes limiting rank factor). According to the cumulative distribution function (CDF) we can conclude that with probability >95% the residual of the rank-2 approximation is 20 dB lower than first singular value; in summary:

$$\text{rank}(H) \lesssim 2 \text{ for GSM air interface ULA of 8 elements}$$  \quad (10)

### Multichannel and interference estimation

Interference is the limiting factor in cellular mobile radio systems, but in general, we do not know neither the channel $H$ nor of of the covariance matrix of the CCI $Q$; hence both have to be estimated using training sequences $X_t$. The full rank ML unstructured estimates $\tilde{H}, \tilde{Q}$ are given by:

$$[\tilde{H}, \tilde{Q}] = \arg \min_{H, Q} \left\{ \ln |Q| + \frac{1}{2} tr \left( (Y - HX)^H \tilde{Q}^{-1} (Y - HX) \right) \right\}$$  \quad (11)

by deriving with respect to the parameters it yields:

$$\tilde{H} = \tilde{R}_{xx} \tilde{R}_{xx}^{-1}, \quad \tilde{Q} = \tilde{R}_{yy} - \tilde{R}_{yx} \tilde{R}_{xx}^{-1} \tilde{R}_{xy}$$  \quad (12)

$\tilde{R}_{xx} = XX^H / N$ is the sample covariance matrix, $\tilde{R}_{yx}$ and $\tilde{R}_{xy}$ are similarly defined. By constraining the rank of $H$ to be $r$, the estimate of the whitened multichannel is:

$$\tilde{H} = \Pi_r Q^{-1/2} H$$  \quad (13)

$\Pi_r$ stands for the projector onto the leading subspace of the symmetric positive definite pencil $(\tilde{H}H^H, \tilde{Q})$, or equivalently $\Pi_r$ spans the subspace associated to the $r$ leading generalized singular values of the pencil.
IV. ADAPTIVE REDUCED-RANK MLSE

Since in practical mobile radio systems the interference is quickly time-varying, the receiver structure has to adapt efficiently the channel estimates, even within the burst. In principle a truncated generalized singular value decomposition (TGGSVD) of $\mathbf{H}$ could be used to compute the metric, in practice the computational requirement for a GSVSD, and its updating, it is too high ($O(p^3)$, where $l = min(M, L)$) for being implemented on-line. The technique proposed here is based on an iterative recursive least squares that tracks the principal whitened subspace. This approach leads to an attractive parallel structure. As the channel and interferences are time-varying, the system allows a fast subspace tracking for the updating of the leading (left) generalized singular vectors.

A classical method to compute the eigenvector associated with the eigenvalue with largest magnitude is the Power Method [9]. Inverse iteration can be used to compute eigenvectors associated with eigenvalues in the interior of the spectrum. A generalization of the Power Method (or Inverse Power Method) that allows to compute more than one eigenvector is Subspace Iteration [9]. Instead of iterating one vector, the method iterates for $r$ vectors simultaneously and obtains approximate eigenvectors corresponding to the $r$ eigenvalues of largest magnitude. The drawback of this approach is that the columns become linearly dependent. This can be solved by an orthogonalization step in each iteration.

The algorithm is summarized below:

**Algorithm**

Loop on bit $f_n^*$

$\mathbf{r}^*(n) = (\mathbf{r}^*(n-1) + (1 - \alpha)\mathbf{f}_n^*)/\sqrt{\alpha}$, $\alpha \leq 1$;

$\left( \begin{array}{c} R_1^{1/2} \end{array}, Q_2^{1/2} \right) = FSR (\mathbf{R}^*(n), f_n)$

(by means of Givens rotations [5]);

$\mathbf{R}^*(n) = \mathbf{L} \mathbf{L}^*$, where $\mathbf{L} = \left[ \begin{array}{cc} \mathbf{L}_1 & 0 \\ \mathbf{A} & \mathbf{L}_2 \end{array} \right]$;

$Q(n) = \mathbf{L}_3 \mathbf{L}_1^*$

for $i = 1 : r$ [performed in parallel]

$\mathbf{L}_3 \mathbf{L}_2^* \mathbf{e}_n = \mathbf{R}_n \mathbf{u}_n(n-1)$;

$\mathbf{b} = \mathbf{R}_n \mathbf{u}_n(n-1)$;

$\mathbf{y} = \mathbf{L}_3^{-1}$; back substitution 1

$\mathbf{z} = \mathbf{L}_3^{-1} \mathbf{y}$; back substitution 2

update of the Graham Schmidt decomposition of $\mathbf{Z}$;

end;

In addition to rank-modularity, the attractive feature of adaptive reduced-rank receiver is that the cost of the rank-$r$ receiver is almost linearly increasing with rank $O(r^3)$. Further, since the floor level of the singular values depends on the amount of noise in the least squares estimate, an iterative approach for short packets is investigated in the next section (it will be shown how the rank order can be adapted, for each user, according to the interference and noise level).

V. EXAMPLES

To evaluate the effectiveness of this solution, some comparative tests have been performed. Figure 3 shows the comparison of the performances (evaluated in terms of BER of uncoded bits) among rank-1, rank-2 and full-rank receivers for varying signal to interference ratio (C/I). The channel $\mathbf{H}$ was almost rank-2 (the power delay angle diagram of the useful signal is illustrated in the lower left corner), receiver adaptation is performed on a burst-by-burst basis and the CCIIs may appear/disappear randomly within the burst. The received signal impinged on a $\lambda/2$-spaced ULA with $M=8$ sensors. A uniform distribution within the range $[-60, 60]$ deg was adopted for the mean DOAs of the interfering users. That is, a 120deg sectorization has been assumed. As far as the angular spread is concerned, a moderate azimuth spread equal to 8deg has been set. The number of burst-symbols turned out not to be sufficient for the adaptation of the full-rank receiver while the reduced-rank has almost always better performances.

An increasing rank approach was evaluated (the rank is increased, starting from rank-1, until a reasonable accuracy has been reached). The results are shown in figure 4 for TU50 channel. The received signal impinged on a $\lambda/2$-spaced ULA with $M=8$ sensors. A uniform distribution within the range $[-60, 60]$ deg was adopted for the mean DOAs of both the useful signal and the interfering users.

VI. CONCLUSIONS

The paper has shown that the low-rank nature of the wireless channel can be effectively exploited by a reduced-rank receiver as it:

1. reduces the variance of the least squares (unconstrained) estimate of the multichannel $\mathbf{H}$ when short data preambles are available,

2. allows the fast-convergence for tracking fast time-varying CCIIs,
3. reduces the computational load,

4. has an efficient rank-modular structure (figure 1) that can be adjusted to the environment.

The principal whitened subspaces are estimated by a recursive least square approach as it leads to a modular and efficient implementation and it allows to track the time varying channels and/or CCIIs. In addition to the rank-modularity, the attractive feature of the adaptive reduced-rank receiver is that the cost rank-\( r \) receiver is almost linearly increasing with rank. However, since no wireless channel is exactly low-rank, no claim of optimality can be made toward the approach proposed in this paper.

REFERENCES


[4] D.Giancola et al. "Space-Time processing for time varying co-channel interference rejection and

channel estimation in GSM/DCS systems” Proc. of VTC ’99


