In this class we will learn how to design the readout electronics for an accelerometer similar to the one realized in Exercise 1. We will analyze the charge amplifier topology, identifying all the noise contributions and their relative weight in the noise budget of the system. Finally, we will study the drawbacks of our simple stage and we will find an alternative readout topology.

**Problem Description and Questions**

You work in a characterization laboratory at a MEMS company, and you have to design an electronic board in order to test a given capacitive accelerometer. The application of the sensor is vibration monitoring on a mechanical engine in a specific frequency range (from 40 Hz to 400 Hz). The MEMS device has a mass of $8 \text{nKg}$, a mechanical stiffness of $5 \text{N/m}$ and a quality factor $Q = 2$. The capacitive sensing is performed with 5 cells of differential parallel-plate capacitors, each one with stators $200 \mu m$ long. The gap between rotor and stators is $2.5 \mu m$ and the process height is $15 \mu m$. The readout of the device is performed through a charge amplifier configuration stage, as the one represented in figure 1. The capacitance $C_p = 10 \text{pF}$ takes into account the effects of capacitive coupling between rotor and substrate, pads and substrate, wire bondings and other causes.

1. Considering the ‘spring softening’ effect with parallel plates polarized at $V_{dd} = \pm 1.8 V$, calculate the value of the feedback capacitor $C_F$ in order to obtain a sensitivity of $6 \text{mV/\hat{g}}$. 
2. Consider now the bias currents of the operational amplifier \((i_{\text{bias}} = 0.5 \mu A)\). Does this leakage affect the behavior of the stage? Modify the topology of the circuit in order to solve this issue.

3. Make a comparison between the device noise in terms of NEAD \(\hat{g}_p \frac{\text{Hz}}{\sqrt{\text{Hz}}}\) and the front-end electronic noise in terms of \(\hat{g}_p \frac{\text{Hz}}{\sqrt{\text{Hz}}}\). For the latter case evaluate three main contributions: the operational amplifier voltage noise \(S_{v,n} = (10 \times \frac{nV}{\sqrt{\text{Hz}}})^2\), the operational amplifier current noise \(S_{i,n} = (0.3 \times \frac{fA}{\sqrt{\text{Hz}}})^2\) and the resistance thermal noise, providing reasonable approximations for the frequency range of interest.

4. Is this kind of readout suitable to perform a measure of the inclination of your smartphone? If not, how can you modify the circuit for this application?

**QUESTION 1**

An easy way to calculate the sensitivity of the system is to decompose it in different transfer functions between some of the most interesting parameters. In fact, it is possible to calculate the force generated by a certain external acceleration, the movement of the rotor induced by this force, the capacitance variation achievable thanks to this movement and finally the voltage at the output of the charge amplifier stage due to the obtained capacitance variation. The sensitivity of the system can be therefore written as:

\[
S = \frac{\Delta V}{\Delta a_{\text{ext}}} = \frac{\Delta x}{\Delta a_{\text{ext}}} \cdot \frac{\Delta C}{\Delta x} \cdot \frac{\Delta V}{\Delta C}
\]

So it is possible to start evaluating each term individually:
• $\Delta x/\Delta a_{ext} \rightarrow$ This first contribution can be in turn divided into two sub-contributions: $\Delta F/\Delta a_{ext}$ and $\Delta x/\Delta F$. From the second principle of dynamics: $F = m \cdot a$. If the acceleration varies of a quantity $\Delta a_{ext}$, the resulting force applied at the seismic mass varies of a quantity $\Delta F = m \cdot \Delta a_{ext}$. We can then conclude that:

$$\frac{\Delta F}{\Delta a_{ext}} = m$$

For what concern the second sub-term $\Delta x/\Delta F$, one can evaluate it starting from the equation of motion of the spring-mass-damper system, repeating the same analysis presented in Exercise 1. Hence, we know that for $\omega < \omega_0$:

$$\frac{\Delta x}{\Delta F} = \frac{1}{m} \cdot \frac{1}{\omega_0^2} = \frac{1}{k}$$

The stiffness that has to be considered in this calculation is the total stiffness of the device, that is composed by the mechanical stiffness, given as a data, and the electrostatic stiffness, due to the presence of electrostatic forces that arises from the voltage difference between rotor and stators. Again from Exercise 1, we know the formula of the electrostatic stiffness for the parallel plates configuration:

$$k_{elec} = -2V_D^2 \frac{C_0}{g^2}$$

We have to compute first the value of the rest capacitance of the accelerometer, that is easily obtained as:

$$C_0 = \frac{\epsilon_0 A}{g} = \frac{\epsilon_0 L_{pp} h N_{pp}}{g} = 53.1 f F$$

So, it results that $k_{elec} = -0.05 N/m$. We can note that in this particular case the effect of the electrostatic forces is quite low and the derived electrostatic stiffness results negligible (two orders of magnitude lower) with respect to the mechanical stiffness of the device. Therefore for the calculations of this problem the total stiffness of the device ($k_{tot}$) will be considered the same as its mechanical stiffness ($k$).

Putting together the two sub-contributions, the $\frac{\Delta x}{\Delta a_{ext}}$ turns out to be:

$$\frac{\Delta x}{\Delta a_{ext}} = \frac{\Delta F}{\Delta a_{ext}} \cdot \frac{\Delta x}{\Delta F} = \frac{m}{k} = \frac{1}{\omega_0^2}$$

• $\Delta C/\Delta x \rightarrow$ This term is easily obtainable starting from the expressions of the two capacitances when a displacement $x$ of the rotor occurs. With the conventions of figure 1 it is possible to write the following expressions:

$$C_1 = \frac{\epsilon_0 A}{g-x} \quad C_2 = \frac{\epsilon_0 A}{g+x}$$
where \( A \) is the total facing area of all parallel plates capacitors, that is \( A = L_{pp} \cdot h \cdot N_{pp} \).

For small displacement \((x \ll g)\), we know that \( \Delta C = C_1 - C_2 \) can be simplified as follow:

\[
\Delta C = 2 \frac{\epsilon_0 A \Delta x}{g^2} = 2 C_0 \frac{\Delta x}{g} \quad \rightarrow \quad \frac{\Delta C}{\Delta x} = 2 \frac{C_0}{g}
\]

- \( \Delta V / \Delta C \): The general expression of the current through a variable capacitance biased by a generic voltage \( V \) is:

\[
i_c = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C \frac{dV}{dt} + V \frac{dC}{dt}
\]

In our circuit topology, we have a DC voltage across the variable capacitance: since \( dV / dt = 0 \), only the second contribution to the current is relevant. Thus, we can calculate the transfer function from the capacitance variation \( \Delta C \) to the voltage at the output of the charge amplifier stage using the circuital model of figure 2.

![Circuital model for the calculation of the transfer function from capacitance variation to output voltage.](image)

Using the properties of the Laplace transform, we can write the expression of the motional current as:

\[
i(s) = sC(s)V_{DD}
\]

This current, thanks to the negative feedback, flows though the feedback capacitance. The output voltage can be written as follows:

\[
V_{out} = - \frac{1}{sC_F}i(s) = - \frac{sC(s)V_{DD}}{sC_F}
\]

So, one can write that:
Once calculated all the required transfer functions, it is possible to find the value of \( C_F \) in order to match the required sensitivity. First of all we can calculate the mechanical sensitivity, the sensitivity of the device only, not including the contribution of the readout electronics:

\[
S_{mech} = \frac{\Delta C}{\Delta a_{ext}} = \frac{\Delta C}{\Delta x} \cdot \frac{\Delta x}{\Delta a_{ext}} = m \cdot \frac{1}{k_{tot}} \cdot \frac{2C_0}{g} = 0.068 \frac{fF}{m/s^2}
\]

We can write the sensitivity, as common for accelerometers, in terms of gravity unities \( \hat{g} \).

Exploiting the relation \( 1 \hat{g} = 9.8 \text{ m/s}^2 \):

\[
S_{mech, \hat{g}} = S_{mech} \cdot 9.8 \text{ m/s}^2 = 0.67 \frac{fF}{\hat{g}}
\]

So, the dimensioning the feedback capacitance is readily obtained:

\[
S_{\hat{g}} = S_{mech, \hat{g}} \cdot \frac{V_{DD}}{C_F} \rightarrow C_F = S_{mech, \hat{g}} \cdot \frac{V_{DD}}{S_{\hat{g}}} = 0.4 \text{ pF}
\]

**QUESTION 2**

With this circuit topology, a bias current would flow into the feedback capacitor resulting in an ramp-like output voltage in time, that will unavoidably clip on the supply voltage. In fact, starting from the fundamental expression of the capacitor:

\[
i = \frac{C}{dC}{dt} \rightarrow V_{out} = \frac{1}{C_F} \int i_{bias} dt = \frac{1}{C_F} i_{bias} \cdot t + \text{const}
\]

So, in this situation the output of the system, after a brief time, is fix and equal to supply voltages: no accelerations can be readout! The solution is to limit the DC gain of the stage.

![Figure 3: Circuit topology with feedback resistor, and Bode diagram of the transfer function between current and output voltage](image)

5
through a low frequency pole introduced by a feedback resistor, as depicted in figure 3. In this case, the sensor has to correctly measure frequencies from 40Hz to 400Hz, so we can fix the pole one decade before the lower limit, at $f_{pole} = 4Hz$. The feedback resistance value results:

$$f_{pole} = \frac{1}{2\pi R_F C_F} \Rightarrow R_F = \frac{1}{2\pi C_F f_{pole}} = 200G\Omega$$

Please keep in mind that we are working **beyond** the pole of the Bode plot of fig. 3: we are still integrating our signal! Adopting this circuit topology, the bias current of the operational amplifier would simply introduce a DC shift of the output level of:

$$\Delta V_{out} = i_{bias} \cdot R_F = 100mV$$

This offset contribution can be easily suppressed high-pass filtering the front-end output or with other techniques.

**QUESTION 3**

**DEVICE NOISE**

The noise source from a device point of view is the Brownian noise due to the fluctuation force, a force related to the random agitation of particles. It is demonstrated that the effect of this force translates in a white noise in terms of force, with a noise power spectral density of:

$$S_{Fn} = 4K_B T b \left[ \frac{N}{Hz} \right]$$

The NEAD parameter, or Noise Equivalent Acceleration Density, represents the noise in terms of acceleration for the device. Exploiting the transfer function between acceleration and force ($F/a = m$), it is possible to calculate the NEAD of the device:

$$NEAD = \sqrt{S_{An}} = \sqrt{S_{Fn} \cdot \left( \frac{1}{m} \right)^2} = \sqrt{\frac{4K_B T \omega_0}{Qm}} = 160.9 \cdot 10^{-6} m/s^2 / \sqrt{Hz}$$

To obtain the NEAD in terms of $\left[ \frac{ \hat{g} }{ \sqrt{Hz} } \right]$:

$$NEAD_{\hat{g}} = \frac{NEAD}{9.8} = 16.41 \frac{\mu \hat{g}}{\sqrt{Hz}}$$

**ELECTRONIC NOISE**

In order to calculate the electronic noise we can exploit the small signal model of the system. As shown in figure 4, in this case the MEMS capacitance is in parallel with the parasitic capacitance, and this parallel is largely dominated by the parasitic capacitance $C_P \rightarrow C_S = C_{MEMS} + C_P \approx C_P$.

Referring to figure 4, it is possible to start evaluating the three main noise contributions:
Figure 4: Small signal model for the calculation of the electronic noise contributions

- **Op-Amp Voltage Noise** → The voltage of the noise generator is directly carried to the virtual ground, generating a current flow through the parasitic capacitance and so a voltage at the output of the stage:

\[
S_{vn, out} = S_{vn} \cdot \left( 1 + \frac{R_F l}{1/sC_p} \right)^2 = S_{vn} \cdot \left( 1 + \frac{sC_p R_F}{1 + sC_F R_F} \right)^2 \left[ \frac{V^2}{Hz} \right]
\]

This transfer function has a zero and a pole. In which point of the transfer function is the system working point located? For working frequencies larger than 40Hz, the approximation \( \omega R_F C_F \gg 1 \) is valid, so the expression can be simplified:

\[
S_{vn, out} = S_{vn} \cdot \left( 1 + \frac{sR_F C_p}{sR_F C_F} \right)^2 = S_{vn} \cdot \left( 1 + \frac{C_p}{C_F} \right)^2 \left[ \frac{V^2}{Hz} \right]
\]

From this result we can note that the noise increases with the decrease of \( C_F \), but the signal-to-noise ratio doesn't change, as the signal also increases by the same factor. To obtain the value for the noise in terms of \( [\hat{g} \sqrt{Hz}] \) it is only needed to divide the obtained value for the sensitivity calculated above in terms of \( [V/\hat{g}] \):

\[
\sqrt{S_{g, vn}} = \frac{S_{vn, out}}{S_g} = 85.07 \frac{\mu g}{\sqrt{Hz}}
\]

- **Op-Amp Current Noise** → The current of the noise generator directly flows into the parallel of the feedback resistance and capacitance, because in this situation the parasitic capacitance has both the plates connected to ground.

\[
S_{in, out} = S_{in} \cdot \left( \frac{R_F}{1 + sC_F R_F} \right)^2 \left[ \frac{V^2}{Hz} \right]
\]
This transfer function is different for the one relative to the voltage noise: here there is only a pole, at the same frequency of the pole of the voltage noise transfer function. This means that, after the pole, this function lowers as the frequency increases. We can evaluate this contributions depending on operating frequency at the center of our frequency-range (i.e. for $f = 220\,\text{Hz}$): the approximation $\omega R_F C_F >> 1$ is clearly valid, so the expression can be simplified and results:

$$S_{\text{in, out}} = S_{\text{in}} \cdot \left(\frac{R_F}{sC_F R_F}\right)^2 = S_{\text{in}} \cdot \left(\frac{1}{sC_F}\right)^2 \left[\frac{V^2}{\text{Hz}}\right]$$

So, as before, we evaluate the noise related to this contribution in terms of $[\frac{\hat{g}}{\sqrt{\text{Hz}}}]$. Because the noise density is frequency-dependent, we can evaluate the noise at the middle of our bandwidth (i.e. for $f = 220\,\text{Hz}$):

$$\sqrt{S_{\text{in}} \cdot \frac{S_{\text{out}}}{S_g}} = 181 \frac{\mu \hat{g}}{\sqrt{\text{Hz}}}$$

In this frequency range, the operational current noise is larger than the voltage noise contribution.

- **Feedback Resistance Thermal Noise** → If we consider for example the current noise generator for the thermal noise of the resistance, this current all flows into the parallel of the feedback capacitance and resistance, generating a voltage at the output of the charge amplifier stage.

$$S_{r\text{in, out}} = \frac{4K_B T}{R_F} \cdot \left(\frac{R_F}{sC_F R_F}\right)^2 \left[\frac{V^2}{\text{Hz}}\right]$$

As before, for the frequency spot of interest:

$$S_{r\text{in, out}} = \frac{4K_B T}{R_F} \cdot \left(\frac{R_F}{sC_F R_F}\right)^2 = \frac{4K_B T}{R_F} \cdot \left(\frac{1}{sC_F}\right)^2 \left[\frac{V^2}{\text{Hz}}\right]$$

This noise contribution has the same shape of the current noise contribution of the operational amplifier and its value in the frequency range of our interest is:

$$\sqrt{S_{r\text{in}} \cdot \frac{S_{\text{out}}}{S_g}} = \sqrt{4K_B T \cdot \left(\frac{1}{\omega C_F}\right) \cdot \frac{1}{S_g}} = 173 \frac{\mu \hat{g}}{\sqrt{\text{Hz}}}$$

This contribution is comparable to the current noise of the operational amplifier.

From the calculations it is clearly visible that the most important noise contributions for the system in this frequency range are current noise of the operational amplifier and feedback resistance thermal noise. This contributions overcome the intrinsic noise of the device, making the system limited by the electronic noise. We can write:
\[ \sqrt{S_{gn,tot}} = \sqrt{S_{gn} + S_{gin} + S_{grn} + NEAD_g} = 265 \frac{\mu g}{\sqrt{Hz}} \]

With this result, it is possible for example to modify the device lowering its quality factor (Q), that up to now has a value of 2, so generating a little bit of overpeaking when an acceleration containing high-frequency components - like a pulse, or a step - arises.

**Question 4**

The accelerometer, combined with the designed electronics, is not suitable to measure DC accelerations (e.g. gravity), consequently though suitable for vibration monitoring (vibrations are AC accelerations) of this exercise, it can’t measure the inclination of a smartphone. Indeed, the low frequency pole introduced in order to avoid the saturation of the charge amplifier stops the integration of signals with frequency lower than 4 Hz. So, the circuit shown before is very useful from a didactic point of view, but it is rarely adopted for MEMS accelerometers when DC signals should be measured. A solution that allows to readout also DC accelerations consists in a high-frequency modulation of the suspended mass, with each of the sensing electrodes kept to virtual ground: in this way a static acceleration signal produces a motional current at \( f_{mod} \). Note that \( f_{mod} \) should be much higher than the resonant frequency of the accelerometer, in order to not excite the device (the transfer function between force and displacement falls down at high frequencies). The output voltages \( V_{out,1} \) and \( V_{out,2} \) of each readout channel are then ideally subtracted from an ideal Instrumental Amplifier (INA), as reported in figure 5.

![Figure 5: High-frequency capacitive readout circuit topology.](image)

In this situation, in the expression of the current flowing in the capacitor:
\[ i_c = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C \frac{dV}{dt} + V \frac{dC}{dt} \]

The \( C \frac{dV}{dt} \) is no more negligible. Supposing a generic sinusoidal capacitance variation at a frequency \( \omega_a \) (this is a generic situation of a sinusoidal external acceleration, we will simplify the problem later for DC accelerations):

\[ C = C_0 + C_a \cdot \sin(\omega_a t) \]

We can thus derive the motional current:

\[ i_c = C \frac{dV}{dt} + V \frac{dC}{dt} = (C_0 + C_a \sin(\omega_a t)) \omega_{\text{mod}} V_{\text{mod}} \cos(\omega_{\text{mod}} t) + C_a \omega_a V_{\text{mod}} \sin(\omega_a t) \sin(\omega_{\text{mod}}) \]

The ratio of the two contributions modulated at \( \omega_{\text{mod}} \) is roughly \( \frac{\omega_{\text{mod}}}{\omega_a} \): so, considering that consumer accelerometers typically measure signals up to 100Hz, with a \( f_{\text{mod}} = 100kHz \) the \( C \frac{dV}{dt} \) term is 1000 times bigger than the \( V \frac{dC}{dt} \) term, so it will dominate in the sum. In the specific case of DC accelerations, \( \omega_a = 0 \) and only \( C \frac{dV}{dt} \) remains.

We can calculate the expression of the signals \( V_{\text{out},1} \) and \( V_{\text{out},2} \):

\[ V_{\text{out},1} = [C_0 + \Delta C] \cdot \omega_{\text{mod}} V_{\text{mod}} \frac{1}{j\omega_{\text{mod}} C_F} \sin(\omega_{\text{mod}} t) \]
\[ V_{\text{out},2} = [C_0 - \Delta C] \cdot \omega_{\text{mod}} V_{\text{mod}} \frac{1}{j\omega_{\text{mod}} C_F} \sin(\omega_{\text{mod}} t) \]

Finally, the INA will perform a subtraction operation, so the output amplitude will be:

\[ |V_{\text{out}}| = |2\Delta C| \cdot \omega_{\text{mod}} \frac{1}{C_F} \]

At the output of the chain, we have a sinusoidal signal whose amplitude is proportional to \( \Delta C \), from which we can recover information about the acceleration to be sensed. Consequently, we can easily obtain the sensitivity expression for this circuit configuration:

\[ \frac{\Delta V_{\text{out}}}{\Delta a} = \frac{\Delta a}{\Delta a} \cdot \frac{\Delta C}{\Delta C} \cdot \frac{\Delta V_{\text{out}}}{\Delta C} = \frac{1}{\omega_0^2} \cdot \frac{C_0}{g} \cdot 2 \frac{V_{\text{mod}}}{C_F} \]

It is also important to note that the modulation of the rotor voltage is useful to move the signal away from electronics flicker noise (neglected for simplicity in previous noise calculations) and from the DC offset (e.g. the offset given by opamp bias currents in question 2), as depicted in figure 6. As seen before, the capacitance variation is multiplied by a sinusoidal wave at \( f_{\text{mod}} \), because this variable voltage is applied on the rotor. On the other hand, the flicker noise and the offset voltage are introduced by the operational amplifier, after the modulation, and so they remain at low frequencies (in other words, we are modulating the rotor voltage, that modulates the current but does not change the transfer function of the electronic noise to the output). Then, demodulating and low-pass filtering, signal is moved back to base-band and low-frequency noise is shifted at high frequency and cut off.
Figure 6: Effect of modulation and demodulation on signal, offset and noise.

So, the weight of the noise contributions is completely changed: as clear in figure 7, the resistor thermal noise and the current noise of the operational amplifier are now totally negligible, and the contribution of $S_{v,n}$ dominates: the overall noise floor is reduced.
Figure 7: Noise contributions of the system. Note the differences between the two different operating frequencies.