E03
Design of a torsional MEMS accelerometer

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In this class we will learn how to design an out of plane (OP) accelerometer with similar performances with respect to the in plane (IP) one designed in Exercise 1. We will study the accelerometer in terms of linearity error, sensitivity and other relevant parameters. Furthermore, we will make a comparison between IP and OP devices in terms of process height spread.

Problem description and questions

You are a young MEMS designer and your supervisor asks you to redesign a high performance accelerometer sensible to out-of-plane (OP) accelerations, after the good results obtained with the in plane (IP) accelerometer. In order to share or just replicate the same electronic circuit used for IP devices, the OP accelerometer ideally needs to target the performances of the IP accelerometers, in particular: $dC_{\text{diff}}/da_{\text{ext}} = 4.1 \frac{F}{\hat{g}}$, $f_0 = 4456\,Hz$, $FSR = 16\hat{g}$. The geometrical parameters, defined by the process or by other constraints, are reported in Table 1. The parallel plates are biased at $V_{DD} = 3V$.

Refer to Figure 1 for a better understanding of the geometry.

1. Find the maximum angle undergone by the accelerometer in operation and the linearity error.

2. Choose a hole pattern in order to obtain a correct release of the seismic mass, respecting the effective density constraints. Calculate the values of the total stiffness and moment of inertia of the system.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E$ 150 GPa</td>
</tr>
<tr>
<td>process thickness</td>
<td>$h$ 24 $\mu$m</td>
</tr>
<tr>
<td>Polysilicon density</td>
<td>$\rho$ 2320 kg/m$^3$</td>
</tr>
<tr>
<td>max effective density</td>
<td>$\rho_{\text{eff, max}}$ 0.85 · 2320 kg/m$^3$</td>
</tr>
<tr>
<td>min effective density</td>
<td>$\rho_{\text{eff, min}}$ 0.5 · 2320 kg/m$^3$</td>
</tr>
<tr>
<td>start point of PP</td>
<td>$x_0$ 10 $\mu$m</td>
</tr>
<tr>
<td>end point of PP</td>
<td>$x_f$ 150 $\mu$m</td>
</tr>
<tr>
<td>mass 1 length</td>
<td>$R_1$ 150 $\mu$m</td>
</tr>
<tr>
<td>mass 2 length</td>
<td>$R_2$ 300 $\mu$m</td>
</tr>
<tr>
<td>gap of PP</td>
<td>$g$ 1.3 $\mu$m</td>
</tr>
<tr>
<td>available area</td>
<td>$s \times L$ 950 $\mu$m × 450 $\mu$m</td>
</tr>
</tbody>
</table>

Table 1: fixed parameters.

Figure 1: sketch of the top view accelerometers disposition inside the module and sketch of the fixed lateral geometry.

3. Given the above mechanical sensitivity, calculate the width of parallel plates.
4. Evaluate the contribution of the parallel plates in terms of electrostatic stiffness and choose the geometry of the springs.
5. Which are the parameters affected by a thickness variation of the process for both the accelerometers (IP and OP)?

**INTRODUCTION**

Why is it almost mandatory to use torsional accelerometers to reveal out-of-plane accelerations?
1) As shown at lesson, the OOP stiffness becomes orders of magnitude larger with respect to the IP stiffness because of the cubic dependence on process height, a parameter you cannot act on by design.
2) To obtain a differential readout with a pure vertical displacement, we would need an electrode on the cap wafer (typically not available in an industrial process).

The torsional accelerometer avoids these problems: while one half approaches the electrode, the other one moves away, see Figure 2.
Then, we are moving from the linear system described by mass-force-stiffness-displacement
to a torsional system described by inertia-moment-torsional stiffness-angle.
Let us introduce some parameters (you can refer to class n.6 for further details):

- **Shear modulus**: it is defined as the ratio of shear stress to the shear strain. The shear modulus in polysilicon is about \( G = 65 \text{GPa} \).
- **Torque**: is defined as the cross product of the lever-arm distance vector and the force vector. Then, the dimension of torque is \( N \cdot m \).
- **Torsional stiffness**: it is involved in the relationship between angle of twist and the applied torque. For our purpose, the simplified formula of a single torsional bar is enough
  \[
  k = G \frac{hw^3}{3l} \left[ \frac{Pa}{m^3} \right] \rightarrow \left[ \frac{N}{m^2} m^3 \right] \rightarrow [N \cdot m]
  \]
- **Moment of inertia**: is the second moment of mass with respect to distance from an axis. Then, the dimension of the inertia is \( kg \cdot m^2 \).

Then, we calculate the inertia for the given OP accelerometer. For the sake of simplicity, we can divide this calculation in two parts: one for the left half of the accelerometer and the second one for the right half (see Figure 1).

\[
I = I_1 + I_2 = \int_{m_1} r^2 dm + \int_{m_2} r^2 dm
\]
Figure 4: sketch of a torsional accelerometer, helpful for moment of inertia calculation.

The mass is not considered as point-like: for more accurate results, we can consider it uniformly distributed along the device.

Observing figure 4, it is clear that in this situation is possible to express the infinitesimal mass element as follows:

\[ dm = shp \cdot dr \]

Elaborating the definition of moment of inertia:

\[ I = \int m_1 r_1^2 shp \cdot dr + \int m_2 r_2^2 shp \cdot dr = \frac{r_1^3}{3} shp + \frac{r_2^3}{3} shp = \frac{r_1^2 m_1 + r_2^2 m_2}{3}. \]

In the specific situation of our accelerometer, \( r_2 = 2 \cdot r_1 \) and consequently \( m_2 \) is twice \( m_1 \). It follows that:

\[ I = 3r_1^2 m_1. \]

To calculate the value of an external torque momentum, \( M_{ext} \), it is convenient to solve the motion equation in the non-inertial reference, which for a torsional system is:

\[ I \ddot{\theta} + b \dot{\theta} + k \theta = M_{ext} \]

Note that in a torsional system the unit of measurement of the damping is \([kg/s \cdot m^2]\).

Being \( \Theta(s) \) the Laplace transformation of the relative angle and \( M_{ext}(s) \) the Laplace transformation of the external torque momentum, one can find the transfer function \( T_{\Theta M}(s) \) as

\[ T_{\Theta M}(s) = \frac{\Theta(s)}{M_{ext}(s)} = \frac{1/I}{s^2 + \frac{b}{I} s + \frac{k}{I}} \]

Writing this equation in terms of \( jw \) and considering that \( \omega_0 = \sqrt{k/I} \) and \( Q = \omega_0 I/b \), we obtain
Evaluating the transfer function modulus at different value of $\omega$, one can find the relation between angle and torque momentum

- **below the resonance frequency $\omega_0$**

\[
\left| T_{\theta M}(j\omega) \right|_{\omega<<\omega_0} = \frac{1}{\omega_0^2} = \frac{1}{k}
\]

this means that when a quasi-stationary torque moment is applied to the seismic mass the angle is governed just by the stiffness constant,

- **at the resonance frequency $\omega_0$**

\[
\left| T_{\theta M}(j\omega) \right|_{\omega=\omega_0} = \frac{1}{\sqrt{\left(\omega_0^2\right)^2}} = \frac{Q}{\omega_0^2} = \frac{Q}{k}
\]

this means that applying a torque moment at the resonance frequency of the device, the angle increases by a factor $Q$ with respect to the quasi-stationary case,

- **above the resonance frequency $\omega_0$**

\[
\left| T_{\theta M}(j\omega) \right|_{\omega>>\omega_0} = \frac{1}{\omega^2}
\]

this means that above the resonance frequency the angle becomes proportional to the inverse of the frequency modulation of the applied torque moment.

**QUESTION 1**

To evaluate the maximum angle reached by the accelerometer, $\theta_{max}$, one can write the angle in terms of applied torque moment on the seismic mass through the transfer function $|T|$, remembering that the accelerometer works at $\omega < \omega_0$,

\[
\theta = M \cdot |T(\omega < \omega_0)| \quad \rightarrow \quad \theta_{max} = \frac{M_{FSR}}{k} = \frac{I M_{FSR}}{I} = \frac{1}{\omega_0^2} \frac{M_{FSR}}{I}
\]

where the inertia of the system is $I = 3r_1^2 m_1$, as reported above, and the torque moment at the full scale range can be calculated, assuming the application point of the inertial force in the mid-point of each semi-structure (i.e. the forces are applied in $r_1/2$ and $r_2/2$), as:

\[
M_{FSR} = M_1 + M_2 = \tilde{r}_1 \times \vec{F}_1 + \tilde{r}_2 \times \vec{F}_2 = -\frac{r_1}{2} m_1 \cdot \text{FSR} \cdot \hat{g} + \frac{r_2}{2} m_2 \cdot \text{FSR} \cdot \hat{g} = -\frac{r_1 m_1}{2} \cdot \text{FSR} \cdot \hat{g} + \frac{r_2 m_2}{2} \cdot \text{FSR} \cdot \hat{g} = -\frac{r_1 m_1 + 4 r_1 m_1}{2} \cdot \text{FSR} \cdot \hat{g}
\]
where $FSR \cdot g$ indicates the full scale range acceleration: the value expressed in gravity units is multiplied by the gravity constant $9.8m/s^2$. Given the parameters and the fixed lateral geometry, the accelerometer undergoes a maximum angle $\theta_{\text{max}} = 6.66 \cdot 10^{-4} \text{ rad}$. Note that the angle is determined by the external acceleration, by the resonance frequency and by the geometry.

To evaluate the error of linearity, $\epsilon_{\text{lin}}$, introduced by the parallel plates configuration, we need to write the real capacitance variation and the linearized capacitance variation at the full scale range, indeed:

$$\epsilon_{\text{lin,FSR}} = \frac{\Delta C_{\text{real,FSR}} - \Delta C_{\text{lin,FSR}}}{\Delta C_{\text{real,FSR}}} \cdot 100$$

where $\Delta C_{\text{real,FSR}}$ is computable as

$$\Delta C_{\text{real,FSR}} = C_2 - C_1 = \epsilon_0 \int_{x_0}^{x_f} \frac{l_{pp}}{g - \tan(\theta)x} dx - \epsilon_0 \int_{x_0}^{x_f} \frac{l_{pp}}{g + \tan(\theta)x} dx$$

For sake of simplicity, we can approximate the capacitance variation obtained with this integral with a ‘standard’ parallel plate that performs a displacement equal to the vertical displacement, $z$, experimented by the medium point of the parallel plate, $x_m$, where

$$x_m = \frac{x_f + x_0}{2} ; \quad z = \tan(\theta)x_m - \theta x_m .$$

Then, we can write

$$\Delta C_{\text{real,FSR}} = \epsilon_0 \frac{A_{pp}}{g - z} - \epsilon_0 \frac{A_{pp}}{g + z} = \epsilon_0 \frac{l_{pp}}{g} \left( \frac{1}{1 - \frac{z}{g}} - \frac{1}{1 + \frac{z}{g}} \right) = C_0 \frac{2z}{1 - \left(\frac{z}{g}\right)^2}$$

and with the small displacement approximation, $z \ll g$, one can obtain the expression for $\Delta C_{\text{lin}}$ as
\[ \Delta C_{lin} = 2C_0 \frac{g}{z} \]

Following the same simplifications presented in Ex01, Question1, we can find the expression for the maximum vertical displacement as a function of the maximum linearity error:

\[
\epsilon_{lin} \frac{C_0}{\left| \frac{2z'}{g} \right|} = 100 \left( \frac{C_0}{\left| \frac{2z'}{g} \right|} - C_0 \frac{2z'}{g} \right) - (100 - \epsilon_{lin}) \left| \frac{1}{1 - \left( \frac{z}{g} \right)^2} \right|_{FSR} = 100
\]

\[
- \left( \frac{z_{FSR}}{g} \right)^2 = \frac{\epsilon_{lin}}{100} \Rightarrow z_{FSR} = g \sqrt{\frac{\epsilon_{lin}}{100}}.
\]

By the definition of \( z_{FSR} \), we can find the relation between the maximum linearity error and the maximum value of the angle

\[
\theta_{max} = \frac{g}{x_m} \sqrt{\frac{\epsilon_{lin}}{100}}.
\]

With the given geometry and parameters, the linearity error results \( \epsilon_{lin} = 0.17\% \).

**QUESTION 2**

As a rule of thumb it can be said that, in absence of other constraints, we want to maximize the stiffness of an accelerometer in order to avoid reliability and stability issues (e.g. adhesion, offset). The expression of the torsional stiffness is:

\[ k = \omega_0^2 I \]

The value of the resonance frequency is fixed, so the higher inertia gives the higher stiffness. The inertia, \( I \), as reported in the introduction, results:

\[ I = 3r_1^2 m_1 = 3r_1^3 \cdot \rho \cdot h \cdot s \]

In order to maximize the inertia, we need to maximize all the free parameters (\( r_1 \) is fixed by the later geometry, \( h \) is fixed by the process): \( s \) and \( \rho \). The maximum value for \( s \) is given by the maximum available area and it results \( s = 950\mu m \). But, how can we change the density of a material? We can change the dimension of the holes pattern on the seismic mass!

Consider a holes pattern with a step size of 10\( \mu m \times 10\mu m \), where the hole is a square with a 4\( \mu m \) side. The effective density results the 84\% of the material density. If the hole side is 7\( \mu m \), the density reaches roughly half of its original value. Then, we can choose the maximum density allowed by this process, perforating the mass as represented in figure 6. The maximum inertia results \( I = 4.49 \cdot 10^{-16} \text{kg} \cdot \text{m}^2 \). Consequently, the maximum value of total stiffness is \( k_{tot} = 3.5 \cdot 10^{-7} \text{N} \cdot \text{m} \). Please note that we are considering our variable MEMS capacitor as a simple parallel plate structure, as the one seen for the in-plane accelerometer exercises. But,
taking a look to the designed structure it is evident that we are in a particular situation: our parallel plate capacitor has a perforated plate (the rotor!). What about fringe fields and other second order effects? Simulation tools come in our help in this kind of situations: we will deeply investigate this configuration in the first CAD exercise.

**QUESTION 3**

The mechanical sensitivity is defined as the capacitance variation per gravity unit of acceleration. Then we can find the needed dimensions of parallel plate to satisfy the target (we are considering the 'vertical displacement approximation')

\[
\frac{dC_{diff}}{da_{ext}} = \frac{\Delta C_{real, FSR}}{FSR} \Rightarrow \frac{dC_{diff}}{da_{ext}} \cdot FSR = \Delta C_{real, FSR} = 64.36 fF
\]

\[
\Delta C_{real, FSR} = \varepsilon_0 \frac{l_{PP} \cdot (x_f - x_0)}{g} \cdot \frac{2 \frac{z_{FSR}}{g}}{1 - \left(\frac{z_{FSR}}{g}\right)^2}
\]

Where \(z_{FSR} = \Theta_{FSR} \cdot x_m\). In order to reach the sensitivity target, the parallel plates length is \(l_{PP} = 865 \mu m\).

**QUESTION 4**

The considered sensor presents a capacitive readout based on parallel plates configuration. To evaluate the effect of electrostatic softening introduced by the electrostatic torque momentum of the parallel plates, one can rewrite the characteristic equation of the spring-mass-damper:

\[
I \ddot{\theta} + b \dot{\theta} + k \theta = M_{tot} \rightarrow I \ddot{\theta} + b \dot{\theta} + k = M_{ext} + M_{elec}
\]

Then, one can calculate the expression for the electrostatic force, \(M_{elec}\) in the case of differential parallel plates configuration and 'vertical displacement approximation'.
\[ M_{elec} = M_{elec,2} + M_{elec,1} = (F_{elec,2} + F_{elec,1}) \cdot x_m = \frac{V_{DD}^2}{2} \left( \frac{\partial C_2}{\partial z} + \frac{\partial C_1}{\partial z} \right) \cdot x_m = \]

\[ = \frac{V_{DD}^2}{2} C_0 \left[ \frac{1}{\frac{z}{g}} - \frac{1}{1 - \frac{z}{g}} \right] \cdot x_m \]

Thanks to the small displacement approximation, \( z \ll g \), the electrostatic torque moment, \( M_{elec} \), can be written in the simplified formula

\[ M_{elec} = \frac{V_{DD}^2}{2} C_0 \cdot \frac{4 \frac{z}{g}}{1 - 4 \left( \frac{z}{g} \right)^2} \cdot x_m = 2 \frac{C_0}{g^2} V_{DD}^2 \cdot z \cdot x_m = 2 \frac{C_0}{g^2} V_{DD}^2 \cdot \theta \cdot x_m \cdot x_m \]

Let’s consider a stationary torque momentum applied to the sensor, in this case the general equation of the motion become

\[ I \ddot{\theta} + b \dot{\theta} + k \theta = M_{ext} + M_{elec} \rightarrow k \theta - M_{elec} = M_{ext} \rightarrow \theta (k_{mech} + k_{elec}) = M_{ext} \]

having defined

\[ k_{elec} = -2 \frac{C_0}{g^2} \Delta V^2 \cdot x_m. \]

In our case \( k_{elec} = -5.3 \cdot 10^{-8} N \cdot m \). Beside, we can find the relation between the total stiffness in operation, \( k_{tot} \), the mechanical stiffness, \( k \), and the electrostatic stiffness, \( k_{elec} \), as

\[ \theta (k + k_{elec}) = M_{ext} \rightarrow k + k_{elec} = \frac{M_{ext}}{\theta} \rightarrow k + k_{elec} = k_{tot} \]

So, it turns out that our mechanical stiffness is \( k = 4 \cdot 10^{-7} N \cdot m \). First of all, let’s keep in mind the geometry presented in figure 4. In order to obtain a correct torsion of the accelerometer, it is necessary to use two springs in parallel configuration, as in figure, resulting in a mechanical torsional stiffness:

\[ k = 2G \frac{w^3 h}{3 l_s} \]

Then, we can select a length for the springs. We want the torsional element to be as long as possible, because, at fixed stiffness, it results in a wider spring (so, as for in-plane devices, less 'over etch' problems). The available dimension is \( L = 950 \mu m \). We have to fit on this space, two springs and relative anchorage to the substrate. Let’s suppose a single anchorage \( 50 \mu m \) longs, the corresponding length for the spring turns out to be \( l_s = 450 \mu m \). Now, we can find the width of the springs as

\[ w_s = \sqrt{\frac{3 l_k}{2 G h}} = 5.6 \mu m \]
QUESTION 5

A typical problem related to the design of MEMS is the variation of the fabrication process. The flow is composed by different steps and each single step can introduce imperfection in the definition of the geometry. Typical issues related to the process are

- variation of the springs width (principally due to the drie etching)
- variation of the thickness (due to the non uniformity in the epitaxial growth)
- variation of the pressure inside the module
- stiction.

It is important, during the design phase, to take into account these issues and to find a way to reduce these unwanted effects.

Suppose now that a problem occurs during the deposition process: the height of the IP structures and of the OP accelerometer is actually 25 $\mu m$ instead of 24 $\mu m$. Which are the parameter affected by this change and how do they change as a function of the thickness?

**IP**

- Of course, the seismic mass is proportional to the thickness.
- The electrostatic stiffness results proportional to the thickness ($C_0(h)$).
- The IP stiffness is proportional to the thickness, then also the total stiffness (electrostatic+mechanical).
- The IP resonance frequency is independent of the thickness: this result rises from the proportionality of the frequency resonance to the total stiffness and its inverse proportionality to the mass. This result is very important because it means that also the performed displacement per applied acceleration is itself independent of $h$.
- The IP differential capacitance variation per gravity unit (i.e. the mechanical sensitivity) is proportional to the thickness. It means that for the same plane geometry, a larger $S_{mech}$ is obtained by a thicker device. If we want repeatable devices in terms of sensitivity, we should ensure a uniform height on the wafer.

**OOP**

- The seismic mass is dependent on the thickness and so the other parameter related to the seismic mass, the moment of inertia.
- The torsional stiffness is proportional to $H$.
- The electrostatic stiffness, in OOP accelerometer, is independent of the process height, because now $C_0$ is independent of $H$. 


• The resonant frequency slightly varies with the height, because the electrostatic stiffness remains constant. But, given that the electrostatic contribution is usually much lower than the mechanical one, we can say that the resonant frequency is relatively constant in presence of height spreads.

• Consequently, for OOP accelerometers, the sensitivity seems to remain roughly constant if a process height fluctuation occurs.