In this class we will learn how to build an electronic oscillator whose frequency-selective element is a MEMS resonator. We will evaluate the dependence of the quality-factor and the motional resistance of a MEMS resonator from temperature, we will discover why a charge-amplifier-based front-end offers better performances than a trans-resistance amplifier, we will learn how to dimension the charge amplifier, and how to design the whole electronic circuit, with particular attention on satisfying Barkhausen criteria of oscillation.

PROBLEM

You work in an circuit design company. You are asked to design an electronic oscillator whose frequency-selective element is a MEMS resonator\(^1\), whose electro-mechanical parameters are listed in Table 1. The minimum capacitance allowed by the integrated process is 200 fF. The supply voltage of the circuit (±\(V_{DD}\)) is ±3.3 V.

We are asked to . . .

1. Calculate the maximum equivalent resistance, \(R_{eq,\text{max}}\), of the resonator, considering the dependence of the quality factor on temperature.

\(^1\)We usually define the resonator as the high-\(Q\) mechanical structure, and the oscillator as the system whose oscillation frequency is set by the resonator.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_r$</td>
<td>32768 Hz</td>
</tr>
<tr>
<td>$m$</td>
<td>0.8 nKg</td>
</tr>
<tr>
<td>$h$</td>
<td>15 µm</td>
</tr>
<tr>
<td>$g$</td>
<td>1.8 µm</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>2000</td>
</tr>
<tr>
<td>$N_{CF}$</td>
<td>70</td>
</tr>
<tr>
<td>$V_{DC}$</td>
<td>5 V</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>$-45^\circ$ C to $+85^\circ$ C</td>
</tr>
</tbody>
</table>

Table 1: Electro-mechanical parameters of the MEMS resonator.

2. Dimension the charge-amplifier-based front-end, used to read out the motional current.

3. Determine whether an additional stage is needed before closing the loop, considering that the maximum displacement amplitude of the proof mass, $x_{a,max}$, is 2 µm. If yes, describe and dimension such a stage.

4. Determine if it is necessary to insert another stage, considering the need to satisfy Barkhausen criteria at the resonance frequency of the resonator. If yes, describe it, choose where to place it and dimension it.

INTRODUCTION

Reference oscillators are ubiquitous elements used in almost any electronics system and constitute a multi-billion dollar market in today’s electronic industry. These oscillators are used for a wide range of applications varying from keeping track of real time, setting clock frequency for digital data transmission, frequency up- and down-conversion in RF transceivers and clocking of logic circuits. For mainstream consumer-type applications, two technology families are distinguished: mechanical and electrical oscillators. Regarding mechanical oscillators for high-end applications (e.g., wireless communication, real-time clocks, high-speed digital interfaces), the frequency-selective element is a mechanical resonator made from quartz. An emerging class of mechanical oscillators is based on MEMS technology. The extraordinary small size, high level of integration, low cost and high volume manufacturing capability that is possible with MEMS appear to open exceptional possibilities for creating miniature-scale precision oscillators at low cost. It can be expected that a MEMS-based oscillator has a superior noise performance and frequency stability compared to electrical oscil-
lators, since the MEMS-based oscillator is based on mechanical resonance exhibiting a high $Q$-factor.

An oscillator consists of a frequency-selective element, which is the mechanical resonator, and a gaining element, which is the feedback amplifier. The feedback or sustaining amplifier is required to sustain the resonance in the frequency selective element. The interface between the resonator and the sustaining amplifier accommodates the transfer of electrical energy into mechanical energy and vice-versa. Various ways of transduction are possible. MEMS oscillators have been demonstrated using a variety of transduction principles, but the most commonly used transductions are piezoelectric and capacitive.

The output signal of an ideal oscillator is a perfectly harmonic signal. Whether it is a sine wave or a square wave, it can be fully described with its fundamental harmonic,

$$v_o(t) = A \cdot \sin(\omega_o t),$$

where $A$ is the oscillation amplitude, and $\omega_o$ is the oscillation frequency.

An oscillator needs to fulfill two oscillation criteria (usually regarded as Barkhausen criteria of oscillation) in order to enable and sustain a stable oscillation. The magnitude of the open loop, $G$, at the oscillation frequency $\omega_o$ and oscillation amplitude $A$, should equal unity:

$$|G(\omega_o)| = 1,$$

while the phase shift in the oscillator loop should be equal to a multiple of $360^\circ$:

$$\angle G(\omega_o) = 360^\circ.$$

The loop gain is defined by the combined transfer of the resonator and the sustaining amplifier. It therefore depends on the amplifier topology being applied. A precise unity-gain loop is not achievable. Any imperfection in both the resonator and or in the sustaining amplifier would make it deviate from 1. In practice, the loop gain is designed to be larger than unity at start-up. Random noise is present in all circuits, and some of that noise will be near the desired oscillation frequency. A loop gain greater than one allows the signal at that frequency to increase exponentially each time around the loop. With a loop gain greater than one, the oscillator will start. With a positive loop gain, in theory, the oscillation amplitude would increase without limit. In practice, the amplitude will increase until the output runs into some limiting factor such as the power supply voltage (the amplifier output runs into the supply rails) or the amplifier output current limits. The limiting reduces the effective gain of the amplifier (the effect is called gain compression), and, in stable conditions, the average loop gain will be one.

**QUESTION 1**

The $Q$-factor of a resonator is an important parameter that determines the frequency accuracy performance of an oscillator. It is defined in different ways. The mechanical $Q$ of the
resonator is defined as the ratio of energy stored in the resonator, $E_{\text{stored}}$, divided by the energy dissipated caused by mechanical damping during one period of resonance, $E_{\text{diss}}$:

$$Q = \frac{2\pi E_{\text{stored}}}{E_{\text{diss}}};$$

in other words, it describes how under-damped a resonator is. A high $Q$ indicates a low rate of energy loss relative to the stored energy of the resonator, i.e., oscillations die out more slowly. Said in other words, the higher is $Q$, the lower is the energy that has to be provided to the structure to sustain its oscillation.

The resonator $Q$-factor can also be derived from a graphical point of view, related with the bandwidth of the resonator's transfer function:

$$Q = \frac{\omega_r}{\omega_1 - \omega_2},$$

where $\omega_1$ and $\omega_2$ are the two frequencies where the motional admittance is $1/\sqrt{2}$ times the admittance at $\omega_r$, as shown in Fig. 1.

There are many reasons to aim for a high resonator $Q$-factor. High resonator $Q$-factors result in low resonator impedance, since $R_m$ is inversely proportional to $Q$. Low resonator impedance allows easier oscillator design to meet oscillation conditions. High $Q$-factors will also result in a low phase noise close to carrier. Furthermore, high-$Q$ resonators are required for good frequency stability of the oscillator.

Figure 1: Graphical analysis on resonator $Q$-factor, from bandwidth considerations.
As mentioned, the value of the quality factor is mainly related to damping phenomena. Indeed, the quality factor can be expressed as

\[ Q = \frac{\omega_r m}{b}. \]

Given a certain geometry, i.e., given \( \omega_r \) and \( m \), the \( Q \)-factor depends on the damping coefficient, \( b \). At the pressure range at which MEMS resonators are usually packaged (1 mbar, or lower), the most relevant damping phenomenon, i.e., the one that basically defines the quality factor, is air damping. In this case, energy loss is caused by collisions between gas molecules and the structure walls while the latter are moving or deforming.

One property of the damping coefficient, hence of the quality factor, consists in the dependence on temperature. As the package is hermetic, the volume inside the package is constant; hence, a temperature change causes a pressure change. The dependence of the quality factor on temperature can be empirically expressed as

\[ Q(T) = \alpha \frac{1}{\sqrt{T}}, \]

where \( \alpha \) is a fitting coefficient that depends on the thermo-mechanical parameters of the resonator. As shown in Fig. 2, this approximation is quite good for a wide temperature range around 300 K.
As we learned in previous classes, the equivalent resistance of the resonator, $R_m$, is directly proportional to the damping coefficient, $b$, hence inversely proportional to $Q$:

$$R_m = \frac{b}{\eta^2} = \frac{1}{\eta^2} \frac{\omega_r m}{Q}.$$ 

The maximum value for $R_m$, $R_{m,\text{max}}$, is found when the quality factor is minimum. And the minimum quality factor, $Q_{\text{min}}$, is found at the maximum temperature, $T_{\text{max}} = 273 \text{ K} + 85 \text{ K} = 358 \text{ K}$. Using the formula given before:

$$\frac{Q(T_1)}{Q(T_2)} = \sqrt{\frac{T_2}{T_1}} \Rightarrow Q(T_{\text{max}}) = Q(T_{\text{room}}) \cdot \sqrt{\frac{T_{\text{room}}}{T_{\text{max}}}} = Q_{\text{min}} = 1831$$

With the calculated minimum $Q$ value, we can evaluate the maximum equivalent resistance, $R_{m,\text{max}}$, of the resonator. First of all, we can evaluate the damping coefficient:

$$b_{\text{max}} = \frac{2\pi f_r m}{Q_{\text{min}}} = 90 \cdot 10^{-9} \text{ N/(m/s)}.$$ 

We can calculate the capacitance variation per unit displacement of the resonator,

$$\frac{\partial C}{\partial x} = \frac{2\epsilon_0 h N_{CF}}{g} = 10 \text{ fF/\mu m},$$

and the transduction coefficient,

$$\eta = V_{DC} \frac{\partial C}{\partial x} = 51 \cdot 10^{-9} \text{ VF/m}.$$ 

The maximum equivalent resistance is thus

$$R_{m,\text{max}} = \frac{b_{\text{max}}}{\eta^2} = 33.8 \text{ M}\Omega.$$ 

This is a typical motional resistance value for MEMS resonators in the 10-100 kHz range. Why did we calculate the maximum equivalent resistance? Because, as we will discover later, if Barkausen criteria are satisfied in this worst-case condition, they will then be always satisfied. Remember: for an oscillator, the lower the motional resistance, the better; the higher the motional resistance, the more critical the design of the electronics.

**QUESTION 2**

Before dimensioning the charge amplifier (CA), i.e., the front-end that senses the capacitance variation at the sense-electrode of the resonator and translates this into a voltage, we will evaluate the reason to choose a CA front-end rather than a trans-resistance amplifier (TRA).²

²We usually define trans-impedance amplifier (TIA) any electronic circuit that senses a current and outputs a voltage; in MEMS world, the sensed current is the motional one, that flows through the sense port. Depending
loop can be closed to the actuation electrode, which will then deliver the suitable voltage that compensates for mechanical losses, ensuring stable oscillation. If the feedback amplifier is properly designed, the motional current will be a sinusoidal wave,

\[ i_m(t) = i_{ma} \sin(\omega_o t), \]

where \( \omega_o \) is the oscillation frequency. We will design the circuit in such a way that the oscillation frequency, \( \omega_o \), will be equal to the resonance frequency of the resonator, \( \omega_r \).

The topology of the circuit is the same for the two cases, as you can see in Fig. 3. The relevant difference consists in the value of the feedback components. The transfer function between the input current and the output voltage of this amplifier can be easily evaluated as

\[ T(s) = \frac{V_{out}(s)}{I_{in}(s)} = -\left( R_F \parallel \frac{1}{sC_F} \right) = -\frac{R_F}{1 + sC_FR_F}. \]

In a trans-resistance amplifier (TRA), the feedback is dominated by the resistance\(^3\). This means that the pole of the feedback network must be at least a decade after the operating frequency, which, in our case, is the resonance frequency of the resonator.

\[ \omega_F \equiv \frac{1}{R_F C_F} \gg \omega_o. \]

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\(^3\)What do we mean with the feedback is dominated by the resistance at the frequencies of interest? We mean that, at the frequencies of interest, the impedance of the feedback resistance is lower than the impedance of the feedback capacitor. Which are the frequencies of interest? It depends from the context. In our example, we know that, at equilibrium, the motional current will have a sinusoidal waveform at the resonance frequency of the resonator.
Figure 4: Magnitude of the transfer function of the front-end electronics as the feedback resistance changes. The feedback capacitance is kept constant, equal to 200 fF.

and the transfer function of the amplifier at resonance can be approximated as

$$T_{TRA}(j\omega_o)\Big|_{\omega=\omega_o} = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}\Big|_{\omega=\omega_o} \approx R_F$$

With this architecture, the working frequency falls within the plateau of the Bode diagram of the magnitude of the transfer function, as shown in Fig. 4. Fig. 5 shows the phase shift, for different $R_F$ values.

On the other hand, with a TCA architecture, the feedback impedance at the operation frequency is dominated by the capacitance. This means that the pole of the circuit is at least a decade before the working frequency,

$$\omega_F = \frac{1}{R_FC_F} \ll \omega_o,$$

and the transfer function, around $\omega_o$, is

$$T_{TCA}(j\omega)\Big|_{\omega=\omega_o} = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}\Big|_{\omega=\omega_o} \approx \frac{1}{j\omega C_F}.$$ 

Why should we choose the TCA? Because the TCA minimizes phase noise of the oscillation signal. What is phase noise? As we saw, the output signal of an ideal oscillator can be expressed as

$$v_o(t) = A\sin(\omega_o t),$$
Figure 5: Phase of the transfer function of the front-end electronics as the feedback resistance change. The feedback capacitance is kept constant, equal to 200 fF.

which is an ideal sine wave, i.e., a perfect clock signal. Unfortunately, due to unavoidable noise sources present in the oscillator, this ideal signal will be corrupted. As usually done in RF applications, this worsening of the spectral purity of an harmonic signal is modeled as phase noise. A noisy sinusoidal signal can be written as

\[ v_o(t) = A \sin(\omega_o t + \phi_n) , \]

where \( \phi_n \) is phase noise in time domain. Due to phase noise, the oscillation frequency is not always exactly equal to \( \omega_o \), but may randomly wander. Phase noise, as all random (statistical) quantities, is usually described with its power spectral density (PSD), \( S_\phi \), which is expressed in rad\(^2\)/Hz. As you will learn in other classes/courses, one can demonstrate that

\[ S_\phi \propto \frac{S_{n,V,\text{out}}(\omega_o)}{v_{\text{out},a}^2}. \]

This means that, in order to minimize phase noise, one should minimize the ratio between the power spectral density, evaluated at one node of the circuit, e.g., the output of the TIA, and amplitude of the oscillation signal at that node. Or, said in other words, one should maximize the signal-to-noise ratio (SNR) density, \( SNR_d \), evaluated at that node:

\[ SNR_d = \frac{v_{\text{out},a}^2}{S_{n,V,\text{out}}(\omega_o)}, \]

or

\[ snr_d = \frac{v_{\text{out},a}}{\sqrt{S_{n,V,\text{out}}(\omega_o)}}, \]
when considering the ratio between the signal amplitude and the square root of the noise power spectral density, expressed in V/√Hz. Please, note that the denominator is a noise power spectral density, not an rms noise value, as in a standard SNR evaluation!

The expression of the signal-to-noise ratio density of a generic TIA can be easily evaluated:

$$\text{snr}_d = \sqrt{\text{SNR}_d} = \frac{v_{out}}{\sqrt{S_{n, V, out}(\omega_o)}} = \frac{i_{ma}|Z_F(\omega_o)|}{\sqrt{4k_BT|Z_F(\omega_o)|^2}} = \frac{i_{ma}\sqrt{R_F}}{\sqrt{4k_BT}}.$$ 

The SNR is proportional to square root of the feedback resistance value, and it is independent from the feedback capacitance value. The highest the feedback resistance, $R_F$, the better. As you can see in Fig. 4, the highest values of resistance are found when using a TCA configuration. The message here is that with a TCA front-end one can easily make the noise of the feedback resistor negligible with other intrinsic noise sources, i.e., the thermo-mechanical noise of the resonator and the thermal noise of the OTA.

In addition, when you choose a TRA, you cannot choose independently the SNR and the gain of the stage, as they both depend on $R_F$; on the other hand, with a TCA, you can independently set the SNR and the gain, as it depends only on $C_F$.

For these reasons, the TCA topology offers better performances in terms on SNR, and this is why a TCA is chosen. As mentioned, the gain of the stage at $\omega_r$ (if $R_F$ is sufficiently high) only depends on the capacitance, that can be set as low possible, so that to maximize the signal amplitude at the output node. The capacitance value, $C_F$, is then limited by technological constraints to 200 fF.

In order to make the circuit operate as a charge amplifier, the feedback pole has to be at least a decade below the operating frequency. A pole frequency exactly one decade below the working frequency requires a feedback resistance equal to

$$R_{F, min} = \frac{1}{2\pi f_r C_F} = 243 \text{ M}\Omega.$$ 

Note that, with such a value, the phase shift introduced by the TCA is

$$\angle G_{TCA} = \angle T\left(j2\pi f_o\right) = 180^\circ - \tan^{-1}\left(2\pi f_o R_{F, min} C_F\right) = 95.7^\circ.$$ 

With a one-decade gap between the pole frequency and the operating frequency, the deviation from the theoretical value of $90^\circ$ is not negligible (6°). Note that a two-decade gap ($R_F = 2.43 \text{ G}\Omega$) would lead to a more acceptable $90.6^\circ$ phase shift. We then choose a 2.43-GΩ $R_F$, that guarantees both a higher SNR and a more accurate phase shift between input current and output voltage. This resistor can be (relatively) easily implemented with an MOS transistor biased in its off region.

Once the charge amplifier is chosen, we can calculate its gain, in order to check if it is enough to compensate the losses of the resonator, which are modeled with its motional resistance
The gain of the TCA at \( f_r \) is

\[
G_{TCA} = \left| T_{TCA} (j2\pi f_0) \right| = \frac{1}{2\pi f_0 C_F} = 24.3 \, \Omega.
\]

This gain, which has the dimensions of an impedance, is lower than the value of the maximum motional resistance\(^4\). This means that the TCA on its own is not able to compensate for mechanical losses. In other words, if we decided to close the loop by connecting the output of the TCA directly with the actuation electrode we would have a loop gain equal to

\[
G_{loop} = \frac{G_{CA}}{R_{m,\text{max}}},
\]

which is lower than 1. The resulting circuit would not have enough loop gain to enable and sustain the oscillation, as we would not satisfy the aforementioned Barkhausen criteria. We must add an additional stage, after the TCA.

How can we ensure a loop gain higher than one at start-up... and a loop gain equal to 1 at equilibrium (for stable oscillation)? A time-variant loop gain value is not achievable with linear circuits! Therefore, some non-linearity has to be introduced. We can add a new stage after the TCA, that should both increase the total resistance of the feedback electronics and to introduce the needed non-linearity in the loop. There are basically two ways to implement this stage: a very high-gain voltage amplifier or a hard-limiter (HL). In both cases, at start-up, this additional stage behaves as a linear amplifier, the loop gain is higher than one (at resonance), and oscillation can exponentially grow up from noise, up to when the output voltage gets higher than the supply rails; when this happens, the output voltage clamps to supply rails, it gets distorted, gain compression is experienced and the overall loop gain is brought down to 1, and stable oscillation is guaranteed.

In our example, we will choose a non-inverting hard-limiter (or comparator), with its threshold set to ground. In this way the sinusoidal signal at the output of the charge amplifier saturates to positive and negative voltage supplies as it crosses the ground potential, generating at the comparator output a square wave with a 50% duty cycle and an amplitude that goes from \(+V_{DD}\) to \(-V_{DD}\). With the insertion of the hard-limiter, assuming a 1000 \((G_{HL})\) small-signal gain of the hard-limiter, the loop gain turns out to be higher than 1 at start-up, because the small-signal gain at resonance of the whole electronic circuit (TCA + HL) is now much higher than mechanical losses:

\[
G_{eln} = G_{TCA} \cdot G_{HL} > R_{m,\text{max}}.
\]

The oscillator designed up to now is shown in Fig. 6.

**QUESTION 3**

Unfortunately, the signal at the output of the hard-limiter cannot be used to directly drive the resonator, because a voltage wave with such an amplitude would violate the small-signal

\(^4\)Note that we will evaluate the motional resistance as the maximum one, as this is the most critical situation for an oscillator.
A high ac amplitude would also cause a movement of the mass which would be too large, and the resonator may then show some mechanical non-linearity; this is why we were asked not to exceed a maximum displacement amplitude of 2 \( \mu m \).

It is thus necessary to reduce the amplitude of the waveform at the hard-limiter output. The square-wave shape of the wave can be maintained, as the MEMS itself will filter all the odd harmonics of the signal but the fundamental one. It is thus possible to implement a de-gain stage in its simplest form: a voltage divider. We will now evaluate the voltage amplitude value that, worst-case, causes the mobile mass to move of \( x_{a,max} \). The transfer function between force and displacement for the MEMS resonator at the resonance frequency is

\[
\frac{X(j\omega_r)}{F(j\omega_r)} = \frac{Q}{k} \rightarrow x_a = \frac{Q}{k} F_a.
\]

If the linear approximation is valid, \( F_a = \eta v_a \), we can write

\[
x_a = \frac{Q}{k} \eta v_a.
\]

Remember that, in last equation, \( v_a \) is the amplitude of the sinusoidal voltage that is applied to the actuation electrode. If the actuation voltage is a square wave, we should consider only the amplitude of the first harmonic, as other harmonics will be filtered out by the sharp MEMS transfer function. Remember that a square wave, \( x_{sq}(t) \), which switches from \(+V\) to \(-V\), can be written as

\[
x_{sq} = \frac{4}{\pi} V \left[ \sin(2\pi f_o t) + \frac{1}{3} \sin(6\pi f_o t) + \frac{1}{5} \sin(10\pi f_o t) + \frac{1}{7} \sin(14\pi f_o t) + \cdots \right].
\]

The situation is depicted in Fig. 7.

Remember that, given a fixed actuation voltage amplitude, the displacement amplitude, \( x_a \), is maximum when quality factor is maximum. Recalling the relationship between \( Q \) and \( T \),
the maximum value of quality factor is evaluated for the minimum value of the temperature range of interest, for $T_{min} = 273 K - 45 K = 228 K$:

$$Q(T_{min}) = Q_{max} = Q(T_{room}) \cdot \sqrt{\frac{T_{room}}{T_{min}}} = 2294$$

By dimensioning the actuation voltage for this worst-case situation, we are guaranteeing that, if the temperature increases, $Q$-factor is reduced, and the displacement is reduced, i.e., we will never exceed the maximum allowable displacement of 2 $\mu$m.

The stiffness of the device, $k$, can be easily calculated as

$$k = (2\pi f_r)^2 m = 33.9 N/m,$$

independent from $Q$ variations.

Once both the maximum quality factor and the spring constant are known, we can dimension the ac voltage that should be delivered to the resonator:

$$v_a = \frac{x_{a,max} k}{Q_{max} \eta} = 573 mV,$$

which corresponds to a square-wave whose amplitude is

$$v_{a,sq} = \frac{v_a}{4} = 449 mV.$$

With this numbers, i.e., with this actuation voltage and the maximum quality factor (i.e., minimum temperature) the proof mass would move of 2 $\mu$m. If temperature increases, the dis-
placement would be, as desired, lower. In addition, the small-signal condition is now satisfied:

\[ \frac{v_a}{4} \ll V_{DC} \rightarrow 0.143 \ll 5 \, \text{V}. \]

Once the maximum signal amplitude has been calculated, we have to dimension the voltage divider in order to obtain the desired signal amplitude. The gain of the stage has to be:

\[ G_{DG} = \frac{v_{a, sq}}{V_{DD}} = 0.14 \]

By arbitrarily choosing one of the two resistors, \( R_{DG,1} \), to be equal to 10 kΩ, the value of the other resistor, \( R_{DG,2} \), turns out to be

\[ R_{DG,2} = R_{DG,1} \frac{1 - G_{DG}}{G_{DG}} = 63.4 \, \text{kΩ}. \]

After this de-gain stage it may be appropriate to put a buffer stage, in order to drive the MEMS with a low impedance driver. The complete circuit designed so far is shown in Fig. 8.

**Figure 8:** The circuit we designed up to now: equivalent RLC circuit of the resonator, the TCA, the hard-limiter, the de-gain stage and the buffer.

**QUESTION 4**

Up to now, we designed the front-end, i.e., the stage that reads out the motional current flowing through the sense stator of our resonator, we designed the hard-limiter, that concurrently increases the loop gain magnitude at start-up and provides the required non-linearity for stable oscillation, we designed a de-gain stage as the amplitude of the square wave at the
hard-limiter output is too big to ensure both linear operation and to comply with \( x_{a,\text{max}} \) requirement.

Is this all? We saw that one of the two the Barkhausen criteria, the one on the magnitude, is satisfied, thanks to the non-linear hard-limiter behavior, that makes it much greater than 1 at start-up and exactly equal to 1 during stable oscillation. But what about the phase shift? Let’s cut the loop at the actuation node. As shown in Fig. 9, at resonance the MEMS is a pure resistor, that does not change the phase of the input signal. The charge amplifier stage, being inverting and being an integrator, introduces a phase shift of 90°, the comparator is a non-inverting stage and it does not introduce a phase shift, as well as the de-gain one. In conclusion, a voltage signal at resonance comes to the end of the loop with a phase shift of 90°. The Barkhausen criterion on the phase of the loop gain is not satisfied at resonance! In this condition, the circuit cannot oscillate! Or, better, the circuit may oscillate if there exist a frequency \( \omega_o \neq \omega_r \) that simultaneously satisfies both criteria of oscillation. As we want the circuit to oscillate at the resonance frequency of our resonator, we need to add another stage in the circuit that introduces an additional \(-90°\) phase shift in the loop. The simplest stage that can provide such an output is an inverting differentiator, as the one shown in Fig. 10.

Where can we place it? The best solution is to place it right after the front-end, where the oscillation signal is a sine-wave that can be phase-shifted with no distortion. If we decided to place it after the de-gain stage, we would differentiate a square-wave, i.e., we would obtain an impulsive waveform\(^5\), that is not a suitable actuation waveform.

\(^5\)The derivative of a square wave is a Dirac-delta train.
The transfer function of the stage of a differentiator as the one shown in Fig. 10 is

\[ T_{\text{DIFF}}(s) = \frac{V_{\text{out,DIF}}(s)}{V_{\text{in,DIF}}(s)} = -\frac{sC_1R_2}{(1 + sC_1R_1)(1 + sC_2R_2)}. \]

If properly dimensioned, the stage introduces one zero in the origin, that is exactly what we want, and two poles at higher\(^6\) frequencies. In this case the derivative of the sinusoidal input wave is again a sinusoidal wave, but with a phase shift of \(-90^\circ\) due to the inverting nature of the stage, and \(90^\circ\) due to the zero in the origin, as shown in Fig. 11.

We have now to dimension the components in a way that the two poles introduced by the stage are at least a decade after the resonance frequency of the device, such that the introduced phase shift is (almost) exactly \(-90^\circ\). In addition, it is convenient to fix the gain of the stage at resonance, \(G_{\text{DIFF}} = |T_{\text{DIFF}}(j\omega_0)|\), to be equal to 1. Arbitrarily choosing a feedback resistance, \(R_2\), of 100 k\(\Omega\), the input capacitance, \(C_1\), turns out to be

\[ G_{\text{DIFF}} = 2\pi f_0 C_1 R_2 = 1 \quad \rightarrow \quad C_1 = \frac{1}{2\pi f_0 R_2} = 48 \text{ pF}. \]

From the requirements on the pole frequencies, we can dimension the remaining components. Remember that the poles have to be placed at a frequency higher that the resonance one. As we understood, it is a common practice to take a two-decades margin. Note that the two pole frequencies are not required to be the same, but there is no reason to put them at different ones. By choosing \(f_{p,\text{DIFF}} = 3.2 \text{ MHz}\) as the pole frequency for both poles of the transfer function, the sizing of the remaining components is easy:

\[ C_2 = \frac{1}{2\pi f_{p,\text{der}} R_2} \approx 0.5 \text{ pF}, \]

and

\[ R_1 = \frac{1}{2\pi f_{p,\text{der}} C_1} \approx 1 \text{ k}\Omega. \]

\(^6\text{Higher than the resonance frequency.}\)
With the just-dimensioned differentiating stage, the design of the oscillator is now complete: both Barkhausen criteria are now satisfied. Figure 12 and 13 report the magnitude and the phase of the loop gain. Figure 14 shows the schematic of our complete circuit, while Fig. 15 reports the waveforms at some nodes of the circuit in a detailed way.

Note that the magnitude of the loop gain at resonance can be predicted as

\[
G_{\text{loop}} = \left| T_{\text{loop}} \left( j2\pi f_o \right) \right| = \frac{1}{R_m} G_{CA} G_{HL} G_{DG} \approx 100,
\]

that corresponds to the peak value of Fig. 12.

We can have a look at the amplitudes of the different signals that can be found alongside the loop. Let’s evaluate them at room temperature. Assuming stable oscillation, we saw that the amplitude of the actuation voltage is

\[
v_a = 573 \text{ mV}.
\]

The displacement amplitude, \( x_a \), is

\[
x_a = \frac{Q_0}{k} \eta = 1.74 \mu\text{m},
\]
which is, in accordance with our design, lower than 2 µm. The capacitance variation amplitude, \( C_a \), is
\[
C_a = x_a \frac{\partial C}{\partial x} = 18 \text{ fF},
\]
a typical number of capacitance variation in such devices, while the motional current flowing through the sense electrode (that is then integrated onto the feedback capacitance of the TCA) is
\[
i_{ma} = x_a (2\pi f_o) \eta = 19 \text{ nA},
\]
which is, of course, the same result as
\[
i_{ma} = \frac{v_a}{R_{m,0}} = 19 \text{ nA},
\]
where \( R_{m,0} \) is the motional resistance evaluated at room temperature, equal to 30 MΩ. The amplitude of the sinusoidal voltage at the output of the TCA is
\[
v_{a,TCA} = i_{ma} G_{TCA} = 450 \text{ mV}.
\]
As the gain of the differentiator at resonance is exactly one, the voltage amplitude at the output of the differentiator is exactly the same as the one at the output of the TCA. The hard-limiter then squares the sine-wave at the supply rails \( \pm V_{DD} \) and the de-gain stage reduces the gain down to
\[
v_{a,sq} = V_{DD} G_{DG} = \frac{v_a}{4} = 450 \text{ mV}.
\]
Note that the equivalent resistance of the feedback amplifier is, as expected,
\[
R_{eq,feedback} = \frac{v_a}{i_{ma}} = R_{m,0} = 30 \text{ MΩ},
\]
equal to the motional resistance of the resonator. To be more precise, $R_{eq_{feedback}} = -30 \text{M}\Omega$, i.e., as expected, the feedback amplifier is exactly compensating for mechanical losses and stable oscillation is guaranteed.
Figure 15: Time domain waveforms of your circuit at start-up (up) and at steady-state (down).