In this class we will learn some issues related to the mechanical design of a vibratory MEMS gyroscope. We will learn how to relate a certain drive-axis displacement amplitude with the reference voltage of the amplitude control loop, how the push-pull drive actuation works, how to correctly evaluate both the drive and the sense spring constant, mass, and resonance frequency, and how to evaluate the mechanical sensitivity of the gyroscope.

**Problem**

We are asked to design a tuning-fork MEMS gyroscope, as the one shown in Fig. 1. The parameters of the sensor are reported in Table 1. Note that data are given per half device. The drive mode is actuated in a push-pull configuration through the set of comb-finger electrodes \( C_{da,1} \) and \( C_{da,2} \), with square waves. Comb fingers for drive motion detection, \( C_{dd,1} \) and \( C_{dd,2} \), are biased at ground through the virtual ground of the electronics front-end, while the rotor is biased at \( V_{DC} = 10 \) V. The whole drive loop is reported in Fig. 2.

We are asked to...

1. Determine the voltage \( V_{ref} \) (the AGC reference voltage) in order to induce a drive displacement amplitude of 7 \( \mu \)m.

2. Evaluate the sensitivity error in presence of a 5% error in the \( \partial C/\partial x \) estimation of the drive-detection electrode.
3. Evaluate the in-phase and anti-phase drive resonance frequencies, clearly explaining which frames and springs are involved in the calculations. Determine the overall spring constant of the sense mode to operate in mode-matched conditions.

4. Size the drive-actuation electrode (choose the number of comb-fingers) targeting a 0.5-V peak-to-peak square-wave actuation voltage.

5. Evaluate the mechanical sensitivity of the gyroscope in terms of differential sense capacitance variation per unit angular rate.

Table 1: Gyroscope parameters. Data are for half device.

<table>
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<th>General</th>
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<tr>
<td>( \varepsilon_0 )</td>
<td>( 8.85 \cdot 10^{-12} \text{ F/m} )</td>
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<tr>
<td>( k_B )</td>
<td>( 1.38 \cdot 10^{-23} \text{ J/K} )</td>
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<tr>
<td>( E )</td>
<td>( 150 \text{ GPa} )</td>
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<td>( h )</td>
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<td>( m_i )</td>
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<td>( N_{CF, dd} )</td>
<td>( 30 )</td>
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<tr>
<td>( g )</td>
<td>( 2 \mu\text{m} )</td>
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<tr>
<td>( L_{fd} )</td>
<td>( 180 \mu\text{m} )</td>
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<td>( w_{fd} )</td>
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<td>( n_{PP} )</td>
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<td>( L_{PP} )</td>
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<td>( 500 \text{ fF} )</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>(-45^\circ\text{C} \text{to} +85^\circ\text{C})</td>
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Figure 1: Sketch of the whole gyroscope.
INTRODUCTION

As we learned in previous classes, a vibratory MEMS gyroscope can be modeled as a resonator along the drive axis and as an accelerometer along the sensing axis. Regarding the drive-axis resonator, its peculiarity consists in the need for a precise displacement amplitude, $x_{da}$, in order to properly control the sensitivity of the angular rate sensor. For this reason, an automatic-gain-control (AGC) loop is always present within the drive loop of a MEMS gyroscope.

Usually, a sine wave of amplitude $v_a$ at the resonance frequency of the drive mode is applied to the driving stator and the rotor is biased with a DC voltage, $V_{DC}$. The stator used to detect the drive displacement is biased at ground through the virtual ground of the drive-detection front-end, as shown in Fig. 3. This 3-ports resonator is called a single-ended resonator.

Remember that, in presence of a square-wave drive actuation at the resonance frequency of a high-$Q$ resonator, one can equivalently describe the actuation signal with its first harmonic only, as other harmonics will be filtered out by the sharp transfer function of the resonator:

$$v_a(t) \approx \frac{4}{\pi} v_{a,sq} \sin(\omega_{rd} t),$$

where $v_{a,sq}$ is the peak (not peak-to-peak) amplitude of the square-wave.

The total electrostatic force applied to the proof mass of a single-ended resonator as the one shown in Fig. 3 is

$$F_{e,d} = F_{e,da} - F_{e,dd} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} v_{da}^2(t) - \frac{1}{2} \frac{\partial C_{dd}}{\partial x} v_{dd}^2(t),$$
Figure 3: Sketch of a single-ended drive configuration for MEMS gyroscopes.

\[
F_{e,d} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} (V_{DC} + v_a \sin(\omega_{rd} t))^2 - \frac{1}{2} \frac{\partial C_{dd}}{\partial x} V_{DC}^2,
\]

which can be re-written as

\[
F_{e,d} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} \left[ |v_a \sin(\omega_{rd} t)|^2 + 2V_{DC} v_a \sin(\omega_{rd} t) + V_{DC}^2 \right] - \frac{1}{2} \frac{\partial C_{dd}}{\partial x} V_{DC}^2,
\]

If the number of comb fingers for the drive-actuation and the drive-detection electrodes is the same,

\[
\frac{\partial C_{da}}{\partial x} = \frac{\partial C_{dd}}{\partial x},
\]

the \(V_{DC}^2\) terms get canceled, and the electrostatic force can be re-written as

\[
F_{e,d} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} v_a^2 + \frac{1}{2} \left( \frac{\partial C_{da}}{\partial x} - \frac{\partial C_{dd}}{\partial x} \right) V_{DC}^2 + \frac{\partial C_{da}}{\partial x} V_{DC} v_a \sin(\omega_{rd} t) + \frac{1}{2} \frac{\partial C_{da}}{\partial x} v_a^2 \cos(2\omega_{rd} t).
\]

The electrostatic force applied to the resonator has three contributions: (i) a DC voltage, (ii) a contribution at \(\omega_{rd}\), (iii) a contribution at \(2\omega_{rd}\). In order to have the \(2\omega_{rd}\) component negligible, we need to guarantee that

\[v_a \ll V_{DC}.
\]
If this condition is satisfied, neglecting the DC term\(^1\),

\[
F_{e,d} = \left[ \frac{\partial C_{da}}{\partial x} V_{DC} \right] \frac{v_a \sin(\omega_{rd} t)}{\eta_{da}} = \eta_{da} v_a \sin(\omega_{rd} t),
\]

where \(\eta_{da}\) is the transduction coefficient of the drive-actuation electrode.

In several situations, this condition is enough to guarantee the proper operation of resonators, e.g., the Tang resonator we studied in previous classes for time-keeping applications. There might be some situations, however, for which this condition is not enough. It may happen, for example, that the resonator has a spurious (un-desired) mode at a frequency close to \(2\omega_{rd}\), a typical situation for for complex mechanical structures such as MEMS gyroscopes. Such a mode might be excited by the spurious drive signal at \(2\omega_{rd}\). This must be avoided.

A commonly-used configuration to get rid of this issue is the so-called *push-pull actuation*, depicted in Fig. 4. Such configuration requires a double number of stators for both the drive actuation and the drive detection. There are two big advantages when using this configuration: (i) the intrinsic elimination of the \(2\omega_{rd}\) component and (ii) the elimination of the small-signal hypothesis on the ac actuation voltage. The implementation of the push-pull actuation consists in the application of two ac (sine-wave or square-wave) voltage signals for

\(^1\)A small DC term would negligibly move (offset) the rest position of the proof mass, with no effects on its dynamic behavior.
the drive actuation. The second one is simply a replica of the first one with a 180° phase delay:

\[ v_{a1}(t) = v_a \sin(\omega rt), \]

and

\[ v_{a2}(t) = -v_a \sin(\omega rt), \]

With a push-pull drive actuation, the total electrostatic force can be expressed as

\[
F_{e,d} = F_{e,da,1} + F_{e,da,2} + F_{e,dd,1} + F_{e,dd,2}
\]

\[
= \frac{1}{2} \frac{\partial C_{da,1}}{\partial x} (V_{DC} + v_a \sin(\omega rt))^2 - \frac{1}{2} \frac{\partial C_{da,2}}{\partial x} (V_{DC} - v_a \sin(\omega rt))^2 + \frac{1}{2} \frac{\partial C_{dd,1}}{\partial x} V_{DC}^2 - \frac{1}{2} \frac{\partial C_{dd,2}}{\partial x} V_{DC}^2.
\]

Assuming that the two actuation electrodes are equal,

\[
\frac{\partial C_{da,1}}{\partial x} = \frac{\partial C_{da,2}}{\partial x} \equiv \frac{\partial C_{da}}{\partial x},
\]

and that the two detection electrodes are equal

\[
\frac{\partial C_{dd,1}}{\partial x} = \frac{\partial C_{dd,2}}{\partial x} \equiv \frac{\partial C_{dd}}{\partial x},
\]

the \( V_{DC}^2 \) terms get canceled, and the electrostatic force can be simplified as

\[
F_{e,d} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} \left[ (V_{DC} + v_a \sin(\omega rt))^2 - (V_{DC} - v_a \sin(\omega rt))^2 \right].
\]

Remembering that \((a + b)^2 - (a - b)^2 = 4ab\), with no assumptions, the expression of the electrostatic force can be greatly simplified:

\[
F_{e,d} = \frac{1}{2} \frac{\partial C_{da}}{\partial x} 4V_{DC}v_a \sin(\omega rt) t = \left[ \frac{2}{\eta_{da}} \frac{\partial C_{da}}{\partial x} V_{DC} \right] v_a \sin(\omega rt) = 2\eta_{da} v_a \sin(\omega rt),
\]

where \( \eta_{da} \) is the transduction coefficient of one drive-actuation electrode. Assuming comb-finger electrodes, the electrostatic force can be re-written as

\[
F_{e,d} = 2\varepsilon_0 \frac{h N_{CF} V_{DC}}{g} v_a \sin(\omega rt).
\]

As anticipated, with a push-pull configuration we obtain two results: (i) an electrostatic force ha a component only at the drive resonance frequency and (ii) the drive actuation signal amplitude, \( v_a \) is not limited by the value of \( V_{DC} \).

Note that if \( N_{CF,\text{push-pull}} = 1/2 N_{CF,\text{single-ended}} \), the overall electrostatic force is equal to the one of the single-ended configuration.
**QUESTION 1**

Independently from the mode of operation of a gyroscope, being it mode-match or in mode-split condition, to keep the value of sensitivity stable, it is necessary to introduce an automatic-gain-control (AGC) loop in the drive oscillator. In this way, we can control the displacement amplitude of the drive-axis motion, keeping it stable both from part-to-part variations and against temperature changes.

Figure 5: Sketch of the AGC loop.

In our example (see Fig. 2), the differential voltage signal taken at the output of the front-end, $V_{out}$, is rectified with a full-wave rectifier, and it is then averaged with a low-pass filter (LPF). The LPF output voltage, $V_D$, is compared with a reference voltage, $V_{ref}$. The error signal is processed, and the loop is then closed. If the loop is properly designed, i.e., if the loop gain is high, $V_D$ is equal to the reference voltage, $V_{ref}$. Indeed, thanks to negative feedback, $V_D$ is forced to be equal to $V_{REF}$:

$$V_D \approx V_{ref}$$

Assuming a constant and well-known gain from $x_{da}$ to $V_D$, controlling $V_{ref}$ is equivalent to controlling the drive displacement amplitude $x_{da}$, as shown in Fig. 5.

Our aim is to calculate the proper $V_{ref}$ that forces $x_{da}$ to be equal to 7 $\mu$m. To do this, we need to evaluate the gain from $x_{da}$ to $V_D$.

The half-structure drive-detection electromechanical transduction factor, $\eta_{dd}$, can be evaluated as

$$\eta_{dd} = V_{DC} \frac{\partial C_{dd}}{\partial x},$$

where $\partial C_{dd}/\partial x$ refers to half device. With the numbers provided in Table 1,

$$\frac{\partial C_{dd}}{\partial x} = \frac{2\varepsilon_0 N_{CEdd} h}{g} = 6.37 \text{fF}/\mu\text{m},$$

and

$$\eta_{dd} = 63.7 \cdot 10^{-9} \text{VF/m}.$$
The motional current amplitude flowing through each drive-detection port is

\[ i_{ma} = 2\eta_{dd}x_{da}\omega_{rd}, \]

where the factor 2 takes into account the second half of the device, i.e., takes into account that the current flowing through the unique drive-detection-1 electrode is twice the one calculated for half structure.

The motional currents flowing through the two drive detection ports, \( i_{m1}(t) = i_{ma}\sin(\omega_{rd}t) \) and \( i_{m2}(t) = -i_{ma}\sin(\omega_{rd}t) \), are integrated in the feedback capacitance of the TCA-based front-end. The output voltage variation amplitudes of the TCA are thus

\[ V_{1,a} = V_{2,a} = i_{ma} \cdot \frac{1}{\omega_{rd}C_F} = \frac{i_{ma}}{\omega_{rd}C_F} \eta_{dd}x_{da} = \frac{V_{DC}}{C_F} \left( 2\frac{\partial C_{dd}}{\partial x} \right) x_{da} = 1.78 \text{ V}. \]

The differential output voltage is thus

\[ V_{out,a} = V_{1,a} - V_{2,a} = 3.57 \text{ V}. \]

The sinusoidal voltage signal \( V_{out} \) is then rectified and averaged. The rectification with a full-wave rectifier has a gain equal to \( \frac{2}{\pi} \). The LPF output is then,

\[ V_D = V_{out,a} \frac{2}{\pi} = \frac{2}{\pi} \frac{V_{DC}}{C_F} \left( 2\frac{\partial C_{dd}}{\partial x} \right) x_{da} = 2.27 \text{ V}. \]

This means, that, in order to have a 7-µm drive displacement amplitude, the AGC reference voltage should be set equal to 2.27 V.

**QUESTION 2**

The capacitance variation per unit displacement in a comb-finger electrode can be estimated as

\[ \frac{\partial C}{\partial x} = \frac{2\epsilon_0 N_{CF} h}{g}. \]

This expression is evaluated using the parallel-plate approximation of the rotor-to-stator capacitor. As learned in previous classes, due to border effects, the capacitance value, hence the variation per unit displacement might differ from theoretical analyses. This leads to errors in the gyroscope sensitivity.

Since

\[ x_{da} = \frac{V_{REF}}{\frac{2}{\pi} \frac{V_{DC}}{C_F} \left( 2\frac{\partial C_{dd}}{\partial x} \right)}, \]

A 5% error in the drive-detection capacitance variation per unit displacement estimation, would lead to a 5% sensitivity error. Said in other words, the gain from \( x_{da} \) to \( V_D \) must be known with great precision to prevent sensitivity errors.
QUESTION 3

Let’s have a look at the gyroscope shown in Fig. 1. We learned in previous classes that a dual-mass decoupled gyroscope presents three fundamental resonance modes: the in-phase drive mode, the anti-phase (drive) mode, and the sense mode. In general, other spurious modes are present. These modes (if excited) may force the gyroscope to oscillate along unwanted directions (e.g., out of plane, rotations). A good gyro design takes care of these other modes; the designer tries to develop the structure in such a way that the resonance frequencies of these spurious modes are as far as possible from (i) the fundamental ones, (ii) all the multiples of the fundamental ones that might be excited by mechanical/electronics non-linearities or by square-wave actuation.

As mentioned, along the drive axis we usually find two modes: the in-phase drive mode and the anti-phase drive mode. In a gyroscope, the drive oscillator keeps the anti-phase mode in stable oscillation at resonance, and the in-phase mode is the undesired one.

IN-PHASE DRIVE MODE

The in-phase drive mode is characterized by the simultaneous motion of the two halves of the structure along the same direction.

- Springs: the displacements performed by the two sets of frames are equal. The tuning fork springs (the black folded springs in the middle of Fig. 1) do not bend. Hence, they do not contribute in the determination of the resonance frequency of this mode. When estimating the spring constant of the in-phase mode, $k_{d,ip}$, the only springs of the geometry that should be taken into account are the drive springs (the green folded springs in Fig. 1):

$$k_{d,ip} = \frac{n_{sd}}{n_{fd}} \frac{E h}{l_{fd}} l_{fd}^3 = 33.33 \text{ N/m},$$

where $n_{sd} = 4$ is the number of springs, and $n_{fd} = 2$ is the number of folds for each spring group.

- Mass: which are the frames that define the mass of this resonant mode? The frames that contribute to the resonant motion! In general, the external, light violet, frames are named drive frames: they move along the drive direction and they are electrically interfaced with the drive stators. Of course, this part of device contributes to the drive frame(s) of the device. In addition, this drive frame is connected to the Coriolis frames (the red inner frame), through a set of springs. These springs are designed in order to drag the sense frame during the driving displacement. For this reason, we can consider the total drive mass, $m_d$, as the sum of the external mass and the inner mass:

$$m_d = m_e + m_l = 4.22 \text{ nkg}.$$ 

We can thus evaluate the in-phase resonance frequency, $f_{rd,ip}$, as

$$f_{rd,ip} = \frac{1}{2\pi} \sqrt{\frac{k_{d,ip}}{m_d}} = 14.14 \text{ kHz}.$$
ANTI-PHASE DRIVE MODE

When the anti-phase drive mode is excited, the two halves of the structure move along opposite directions.

- Springs: the tuning fork takes clearly part into the overall drive stiffness. How does the tuning fork contribute to the drive stiffness? During the anti-phase mode, the two ends of the tuning fork perform the same displacement and, thank to the action reaction principle, the central point of the tuning fork spring is permanently in a steady position. Then, we can consider this point as virtually anchored. This is the reason behind declaring the parameter list for half device (Table 1): in this way, we have the right number of tuning fork folds that contribute to the drive motion. The total drive stiffness, \( k_d \), is the sum of the stiffness of the drive springs and of the tuning fork

\[
k_d = k_{d,ap} = k_{ip} + k_{tf} = k_{ip} + \frac{n_{stf}}{n_{tf}} E h \left( \frac{w_{tf}}{l_{tf}} \right)^3 = 62.13 \text{ N/m.}
\]

where \( n_{sd} = n_{fd} = 2 \) is the number of folds for each spring group.

- Mass: the mass is again the sum of the external and the internal frames

\[
m_d = m_e + m_i = 4.22 \text{ nkg.}
\]

The drive resonance frequency is thus

\[
f_{rd} = \frac{1}{2\pi} \sqrt{\frac{k_d}{m_d}} = 19.31 \text{ kHz.}
\]

For this geometrical configuration, the in-phase resonance frequency is lower than the anti-phase resonance frequency. The difference between the two modes is around 5 kHz.

SENSE MODE

In mode-matched operation, the drive resonance frequency, \( \omega_{rd} \), and the sense resonance frequency, \( \omega_{rs} \), are equal. The sense-axis spring constant, \( k_s \), can be thus evaluated as

\[
k_s = m_s (2\pi f_{rs})^2 = 32.68 \text{ N/m.}
\]

Note that, for this calculation, we used only the sense mass, \( m_s = m_i \), equal to the one of the inner frame.

QUESTION 4

A 0.5-V peak-to-peak square-wave driving corresponds to a sine-wave whose amplitude is

\[
v_a = \frac{4}{\pi} \frac{v_{a, sq}}{2} = 318 \text{ mV.}
\]
The drive-mode quality factor, $Q_d$, can be estimated as

$$Q_d = \sqrt{\frac{k_d m_d}{b_d}} = 10240.$$ 

Since

$$x_{da} = \frac{Q_d}{k_d} 2 \eta_{da} v_a,$$

where the factor 2 takes into account the push-pull actuation, the target drive-actuation transduction coefficient is

$$\eta_{da} = \frac{1}{2} \frac{x_{da}}{Q_d v_a} = 66.7 \times 10^{-9} \text{VF/m}.$$ 

Hence,

$$\frac{\partial C_{da}}{\partial x} = \eta_{da} V_{DC} = 6.7 \text{fF}/\mu\text{m}.$$ 

This number refers to a single drive-actuation electrode, for half structure. The number of required comb fingers is then

$$N_{CF,da} = \frac{\partial C_{da}}{\partial x} \frac{h}{2 \varepsilon_0 g} = 31,$$

for half structure, for each drive-actuation electrode.

**QUESTION 5**

We want now to find the mechanical sensitivity of the gyroscope, $S$, defined as the differential sense capacitance variation per unit angular rate,

$$S = \frac{\partial C_{s,\text{diff}}}{\partial \Omega}.$$ 

Since the differential capacitance variation is twice the single-ended one,

$$C_{s,\text{diff}} = C_{s,1} - C_{s,2} = 2 C_s,$$

where $C_s$ is the capacitance variation of one sense port, and splitting the capacitance variation per unit angular rate as

$$\frac{\partial C_s}{\partial \Omega} = \frac{\partial C_s}{\partial y} \frac{\partial y}{\partial \Omega},$$

where $\partial C_s/\partial y$ is the capacitance variation per unit displacement of one sense port, and $\partial y/\partial \Omega$ is the sense displacement amplitude variation per unit angular rate, the sensitivity can be rewritten as

$$S = \frac{\partial C_{s,\text{diff}}}{\partial \Omega} = 2 \frac{\partial C_s}{\partial y} \frac{\partial y}{\partial \Omega}. $$
where the factor 2 takes into account the differential readout. The capacitance variation per unit displacement of one sense port, $\partial C_s/\partial y$, can be expressed as

$$\frac{\partial C_s}{\partial y} = \frac{C_s}{g} = \frac{\epsilon_0 (2N_{PP}) L_{PP} h}{g^2} = 106 \text{ fF/\mu m},$$

where the factor 2 takes into account the two halves of the device. The sense displacement amplitude induced by the Coriolis force, $y_a$, is

$$y_a = \frac{Q_s}{k_s} F_{ca} = \frac{Q_s}{k_s} 2m_s \omega_{rd} x_{da} \Omega = \frac{2Q_s}{\omega_{fs}} m_s \frac{\omega_{rd}}{k_s} x_{da} \Omega = \frac{2Q_s}{\omega_{fs}} \omega_{fs} \omega_{rd} x_{da} \Omega.$$

For mode-matched operation, the drive- and sense-axes resonance frequencies are equal. Hence:

$$y_a = \frac{2Q_s}{\omega_{fs}} \omega_{rd} x_{da} \Omega \approx \frac{x_{da}}{\Delta \omega_{BW} \Omega},$$

where $\Delta \omega_{BW}$ is the sense-axis bandwidth:

$$\Delta \omega_{BW} = \frac{\omega_{fs}}{2Q_s}.$$

Hence,

$$\frac{\partial y}{\partial \Omega} = \frac{x_{da}}{\Delta \omega_{BW}}.$$

The sense-axis quality factor is

$$Q_s = \sqrt{k_s m_s} = \frac{(2\pi f_{fs}) m_s}{k_s} = 2690.$$

The sense-axis bandwidth is thus

$$\Delta \omega_{BW} = \frac{\omega_{fs}}{2Q_s} = 22.52 \text{ rad/s} = 3.5 \text{ Hz}.$$

The corresponding variation of the Coriolis frame position per unit angular rate is

$$\frac{\partial y}{\partial \Omega} = \frac{x_{da}}{\Delta \omega_{BW}} = \frac{x_{da}}{3.5 \text{ Hz}} = \frac{310 \text{ nm}}{(\text{rad/s})},$$

that can be rewritten as

$$\frac{\partial y}{\partial \Omega} = \frac{310 \text{ nm}}{360 \text{ dps}} \frac{\text{360 dps}}{2\pi \text{ rad/s}} = 5.4 \text{ pm/dps}.$$

The total capacitance variation per unit angular rate is thus

$$S = 2 \frac{\partial C_s}{\partial y} \frac{\partial y}{\partial \Omega} = 66 \text{ fF/(rad/s)} = 1.15 \text{ fF/dps}.$$