Problem Description and Questions

You are a young MEMS designer working for a company that is focusing its research interest in alternative technologies for the sensing of magnetic field. Particular effort is put on the development of Lorentz-Force based MEMS Magnetometers. Your task is to optimize the mechanical design of a prototype device sensible to the out-of-plane magnetic field, in order to have an estimate of its performance with respect to commercial products fabricated in other technologies. The magnetometer, shown in figure 1, is a tuning-fork structure exploiting off-resonance operation to enhance the bandwidth and current recirculation to improve the sensitivity. The most relevant parameters of the device are given in table 1.

1. The required bandwidth for our device application is at least $BW = 50\, \text{Hz}$. Assuming off-resonance operation, which is the minimum mismatch we have to use in order to satisfy the bandwidth requirement? Choose a suitable value for the frequency mismatch: in this condition, calculate the mechanical differential sensitivity of the magnetometer, expressed in $[F/T]$.

2. Supposing to operate your device at resonance, is the bandwidth requirement satisfied?

3. The device has to be immune to external accelerations: in particular, the mechanical sensitivity should not vary more than 1% if an input DC acceleration of 32g occurs. Is this requirement satisfied?
Figure 1: Structure of the Lorentz-Force based MEMS magnetometer sensible to the out-of-plane magnetic field. The current recirculates in separated loops into the structure in order to enhance the Lorentz-Force, and so the sensitivity, for the same external magnetic field.

<table>
<thead>
<tr>
<th>Device Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>anti-phase resonance frequency</td>
<td>$f_0$</td>
</tr>
<tr>
<td>in-phase resonance frequency</td>
<td>$f_{0,ph}$</td>
</tr>
<tr>
<td>spring length</td>
<td>$L$</td>
</tr>
<tr>
<td>rest gap</td>
<td>$g$</td>
</tr>
<tr>
<td>process thickness</td>
<td>$H$</td>
</tr>
<tr>
<td>half-structure stiffness</td>
<td>$k_{1/2}$</td>
</tr>
<tr>
<td>current loop number</td>
<td>$N_{loop}$</td>
</tr>
<tr>
<td>parallel-plate cells</td>
<td>$N_{PP}$</td>
</tr>
<tr>
<td>parallel-plate length</td>
<td>$L_{PP}$</td>
</tr>
<tr>
<td>sealing pressure</td>
<td>$P_0$</td>
</tr>
<tr>
<td>damping coefficient for unit area</td>
<td>$b_{area}$</td>
</tr>
<tr>
<td>drive current</td>
<td>$i_{MEMS,rms}$</td>
</tr>
</tbody>
</table>

Table 1: relevant parameters of the MEMS magnetometer
QUESTION 1

The most suitable application for this kind of sensor is the consumer application. For this field the required bandwidth is usually $BW = 50\text{Hz}$, due to the variability of magnetic field coupled to human movement of the device containing the sensor. Off-resonance operation consists in the driving of the magnetometer with a current at a frequency slightly lower than the mechanical resonance frequency of the structure. In this way the bandwidth turns out to be no more dependent on the quality factor of the device, as shown in figure 2 it is limited by the $\pm 3\text{dB}$ value of the transfer function with respect to its value at the operation frequency. Usually the bandwidth is sufficiently high to allow its limitation by means of an electronic filtering at the end of the electronics chain.

![Zoomed region](image)

Figure 2: Off-resonance operation and related bandwidth.

As a rule of thumb, for a certain mismatch we can consider an achievable maximum bandwidth of about $1/3$ of the mismatch value. Therefore for the case of interest the minimum frequency mismatch that has to be taken into account is $BW \cdot 3 = 150\text{Hz}$. To take a safety margin without sacrificing the sensitivity too much we can choose a mismatch of $\Delta f = 200\text{Hz}$.

In this condition, the expression for the differential mechanical sensitivity can be easily written step by step:

$$S_{mech} = \frac{\Delta C}{\Delta B} = \frac{\Delta F}{\Delta B} \cdot \frac{\Delta x}{\Delta F} \cdot \frac{\Delta C}{\Delta x}$$
• The first contribution is related to the Lorentz-Force acting on the structure for a certain external magnetic field. The well-known expression of this force in a conductive wire is:

\[ \vec{F} = iL \times \vec{B} \]

Where \( i \) is the current flowing in the conductor of length \( L \). Please note that the verse of the current flow has been chosen in order to generate an anti-phase displacement of the two masses, as clear in figure 1. Furthermore, the considered device exploits multiple loops for current recirculation: in this way, at fixed driving current, the Lorentz force is amplified by a factor equal to the number of loops:

\[ \frac{\Delta F}{\Delta B} = iLN_{\text{loop}} \]

• The second contribution is related to the achieved displacement for a given force. It can be computed through the transfer function of the MEMS device, taking into account the off-resonance operation and the associated effective quality factor (\( Q_{\text{eff}} \)):

\[ \frac{\Delta x}{\Delta F} = \frac{1}{2} Q_{\text{eff}} \left( \frac{1}{k_{1/2}} \right) \]

\[ Q_{\text{eff}} = \frac{f_0}{2\Delta f} = 45 \]

The factor \( 1/2 \) is related to the distributed nature of the Lorentz-Force acting on the springs. The force is indeed not focused on the central point of the springs as in an accelerometer, but it is distributed overall them. This is equivalent to have a focused force in the center of the springs of a value that is approximately one half of the value of the overall force, causing half the displacement. Please also note that, as usual for tuning fork structures, we are considering the half-structure stiffness \( k_{1/2} \) for the computation of the anti-phase displacement.

• The third contribution is related to the capacitance variation per unit displacement:

\[ \frac{\Delta C}{\Delta x} = \frac{2C_0}{g} \]

where the factor 2 takes into account that we are considering the differential capacitance variation.

We are now ready to write the overall expression of the mechanical sensitivity in terms of \( \frac{\vec{F}}{F} \):

\[ S_{\text{mech}} = \frac{\Delta C}{\Delta B} = iLN_{\text{loop}} \cdot \frac{1}{2} Q_{\text{eff}} \left( \frac{1}{k_{1/2}} \right) \cdot \frac{2C_0}{g} \]

The only missing data is the rest capacitance \( C_0 \), whose calculation is quite straightforward:

\[ C_0 = N_{pp} \cdot \frac{e_0 HL_{pp}}{g} = 520 \, \text{fF} \]
Consequently, the mechanical sensitivity turns out to be:

\[ S_{mech} = 257 \frac{F}{T} \]

**QUESTION 2**

If a MEMS device is operated at its resonance frequency the bandwidth turns out to be inversely dependent on its quality factor through the relation:

\[ BW_{-3dB} = \frac{f_0}{2Q} \]

Starting from the \( b_{area} \) value at 1 mbar, we can evaluate the damping coefficient \( b \) of our device:

\[ A_{1P} = L_{pp} \cdot H \quad \rightarrow \quad b = b_{area} \cdot 2 \cdot A_{1P} \cdot N_{pp} = 0.0845 \frac{\mu N}{m/s} \]

Then we can evaluate the quality factor of the device:

\[ Q = \frac{2 \cdot k_{1/2}}{\omega_0 b} = 12560 \]

Please note that we are using the stiffness of the full structure (\( 2 \cdot k_{1/2} \)) or, equivalently, we are considering half structure and consequently we are dividing the total damping coefficient for a factor 2. The \(-3dB\) mechanical bandwidth results:

\[ BW_{-3dB} = \frac{f_0}{2Q} = 0.7 \text{Hz} \]

Thus, supposing to operate the magnetometer at resonance, the bandwidth requirement is not respected at all. Please note that, in this situation, we can broaden the bandwidth simply increasing the damping coefficient; on the other hand, acting in this way, we are worsening the thermomechanical noise of our device. The off-resonance operation seen in Question 1 prevents this kind of trade-off, as \( Q_{eff} \) is no more a function of \( b \).

**QUESTION 3**

Rejection of external accelerations is a key parameter for many kind of MEMS sensors: if the device is not an accelerometer, an external acceleration shouldn't produce an output signal variation. Considering a DC acceleration, the force acting on the mass generate a DC in-phase displacement of our structure. The sinusoidal anti-phase displacement generated by the Lorentz force is superimposed to this in phase "offset"; the capacitance variation for unit anti-phase displacement should be calculated in this new working point.

For a better understanding of the problem, we can start considering a single differential capacitance as the one in figure 3, with a DC offset due to an external acceleration indicated as \( x_{acc} \).
Figure 3: New working point considering external acceleration effects on a single differential capacitance.

\[ C_{diff} = C_2 - C_1 = \frac{\varepsilon_0 A}{g - x_{acc} - x} - \frac{\varepsilon_0 A}{g + x_{acc} + x} = \frac{\varepsilon_0 A}{g - x_{acc}} \left(1 - \frac{x}{g - x_{acc}}\right) - \frac{\varepsilon_0 A}{g + x_{acc}} \left(1 + \frac{x}{g + x_{acc}}\right) \]

Linearizing for small displacements (i.e. \( x < < g - x_{acc} \)):

\[ C_{diff} \approx \Delta C_0 + \frac{\varepsilon_0 A}{(g - x_{acc})^2} \cdot x + \frac{\varepsilon_0 A}{(g + x_{acc})^2} \cdot x \]

Where \( \Delta C_0 \) is the difference between the two mismatched rest capacitances, not dependent on the displacement. In other words, we can consider this situation as a differential readout scheme with two different rest gaps \( g_1 = x + x_{acc} \) and \( g_2 = x - x_{acc} \) and then two different rest capacitances \( C_1 = \frac{\varepsilon_0 A}{g_1} \) and \( C_2 = \frac{\varepsilon_0 A}{g_2} \). Thus, the differential capacitance variation per unit displacement can be expressed as:

\[ \frac{dC}{dx} = \frac{\varepsilon_0 A}{(g - x_{acc})^2} + \frac{\varepsilon_0 A}{(g + x_{acc})^2} = \frac{C_2 - C_1}{g_2 - g_1} \]

It is evident that considering an ideal differential capacitance, with no external accelerations or offsets \( g_1 = g_2 = g \), we find back the well known expression \( \frac{dC}{dx} = 2 \frac{C_0}{g} \).

So the differential capacitance variation per unit displacement is different from the ideal one, and consequently the last term of the sensitivity expression derived in Question 1 is changed: to quantify this unwanted sensitivity variation we have to consider now our specific situation. First of all, we can compute the DC displacement induced by the external acceleration. We can use the same transfer function used in the accelerometer classes, considering the in-phase resonant frequency (the one excited by an external acceleration):

\[ x_{acc} = \frac{1}{(\omega_{0,ph})^2} \cdot a_{ext} = 80nm \]

This is quite a big displacement compared with the one induced by a magnetic field of 3mT (a typical FSR value):
\[ x_{\text{FSR}} = iLB_{\text{FSR}} \frac{Q_{\text{eff}}}{2k_{1/2}} \cdot N_{\text{loop}} = 1 \text{nm} \]

A quick clarification on FSR: note that the full scale range is defined as the maximum signal that our system **has to measure** and in the case of Lorentz magnetometer it is fixed at some \( mT \) because, in typical applications, the sensor should detect the earth magnetic field (typically of some tens of \( \mu T \)) but on the other hand it has to correctly identify huge magnetic disturbances (e.g. in a smartphone) in order to filter them out. Note that the full scale is fixed by the application, not limited by the device (as in the accelerometer of Exercise 1).

Returning back to our problem, in order to understand the effect of this DC displacement in our case we have to analyze what happens in our 10 parallel plates cells, 5 for each of the two masses coupled by a tuning fork spring. At a glance, the situation could appear a little tricky, but we can schematize our structure as in figure 4.

We have to consider what happen for each of the four capacitors represented: then we can simply multiply each capacitance variation for \( \frac{N_{pp}}{2} \) in order to consider every parallel plate cell in the structure.

For the first capacitance the anti-phase motion due to the Lorentz force reduces the gap, while the in-phase displacement given by the external acceleration increase it, resulting in:

\[ C_1 = \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{g + x_{\text{acc}} - x} \rightarrow \frac{dC_1}{dx} \sim \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{(g + x_{\text{acc}})^2} \]

We can follow the same approach for the other capacitors:

\[ C_2 = \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{g - x_{\text{acc}} - x} \rightarrow \frac{dC_2}{dx} \sim \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{(g - x_{\text{acc}})^2} \]

\[ C_3 = \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{g - x_{\text{acc}} + x} \rightarrow \frac{dC_3}{dx} \sim \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{(g - x_{\text{acc}})^2} \]

\[ C_4 = \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{g + x_{\text{acc}} + x} \rightarrow \frac{dC_4}{dx} \sim \frac{N_{pp}}{2} \frac{\varepsilon_0 A}{(g + x_{\text{acc}})^2} \]

In order to make a correct differential readout of the Lorentz force signal, the capacitors are connected as in figure: the sum \( \Delta C_1 + \Delta C_4 \) will be subtracted from \( \Delta C_2 + \Delta C_3 \), after a proper capacitance to voltage conversion given by the front end electronics (next class topic). So, the total differential capacitance variation per unit displacement is the following:

\[ \frac{dC_{\text{tot,acc}}}{dx} = N_{pp} \frac{\varepsilon_0 A}{(g + x_{\text{acc}})^2} + N_{pp} \frac{\varepsilon_0 A}{(g - x_{\text{acc}})^2} \]

Finally, we can calculate our sensitivity percentage variation, that will be equal to the \( \frac{dC}{dx} \) percentage variation:

\[ \Delta S_0 = \frac{S_{\text{mech}} - S_{\text{mech,acc}}}{S_{\text{mech}}} \cdot 100 = \frac{2C_0}{g} \frac{dC_{\text{tot,acc}}}{dx} \cdot 100 = 0.85\% \]

Thus, we can conclude that the requirement is satisfied and our magnetometer rejects quite good also relatively big external accelerations.
Figure 4: Scheme used for the evaluation of DC acceleration effects on our tuning fork structure.